Department of Geodetic Science

BASIC RESEARCH AND DATA ANALYSIS FOR THE NATIONAL GEODETIC SATELLITE PROGRAM

Ninth Semiannual Status Report

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Washington, D. C. 20546

The Ohio State University
Research Foundation
Columbus, Ohio 43212
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PREFACE

This project is under the supervision of Professor Ivan I. Mueller, Department of Geodetic Science, OSU, and it is under the technical direction of Mr. Jerome D. Rosenberg, Deputy Director, Communications Programs, OSSA, NASA Headquarters, Washington, D. C. The contract is administered by the Office of University Affairs, NASA, Washington, D. C. 20546.

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1. STATEMENT OF WORK

The statement of work for this project includes data analysis and supporting research in connection with the following broad objectives:

(1) Provide a precise and accurate geometric description of the earth's surface.

(2) Provide a precise and accurate mathematical description of the earth's gravitational field.

(3) Determine time variations of the geometry of the ocean surface, the solid earth, the gravity field, and other geophysical parameters.

2. ACCOMPLISHMENTS DURING THE REPORT PERIOD

2.1 Adjustment of BC-4 Worldwide Geometric Satellite Triangulation Net

During this reporting period, NOAA began the shipments to the Data Center of optical observations from the BC-4 worldwide network. This data is in two different forms and is referred to as Type I and Type II data. The Type I data gives satellite image plate coordinates for every satellite image on a plate. Along with this are the camera calibration parameters and a covariance matrix. The Type II data is the result of a polynomial fit to the plate images, which gives seven ficticious images on each plate. These ficticious images are selected at the same time for all stations observing the satellite so that for a particular event, the seven satellite images are simultaneous for all camera stations observing. The term "event", as used by NOAA, refers to a series of images on camera plates; there can be as many as four stations observing, and as many as 400 Type I images or 7 Type II images on each plate. All of this data comprises one event.

Due to the fact that all Type II data is simultaneous, NOAA used Greenwich Hour Angle and Declination as observations for each image. In this way, it was not necessary to record the time of observation. Because of the correlation among the ficticious images on a single plate, a 14 x 14 variance-covariance matrix is given for each set of plate data.

The correlation between different satellite images cannot be taken into consideration in our existing optical adjustment program. In order to test the Type II data, we neglected the correlation between the images on a small sample of the data, generated a ficticious time and converted the GHA's to Right Ascensions, and then input this data into our existing adjustment program. The results showed that the observations were not Greenwich Hour Angle and Declination. A telephone call to NOAA in Rockville, Maryland verified that the observations were not GHA and Declination, but actually Azimuth and Elevation with respect to the origin station, Aberdeen. NOAA then sent a letter describing how we could convert this data to GHA and δ. This conversion was made in our program and used with the old data tapes until NOAA actually sent the correct tapes.

When the correct Type II data tapes finally became available (in December) we had already written and had operational a computer program that would read the data (without correlation) from the original tapes and punch this data (Greenwich Hour Angle and Declination) on cards for input into our existing adjustment program. Changes were also made in the existing adjustment program to accept the Greenwich Hour Angle directly without converting to Right Ascension as required originally. If we neglect the correlation between images we could process all of the data immediately.

Because of the correlation between the images on each BC-4 plate, it became necessary to write a new computer program to process these observations. In the existing program the normal equations were formed one event at a time, but in that case an event was all stations observing one satellite point. It is now necessary to form the normal equations using seven satellite points at once, and this required major changes because now the seven images from each plate and their associated 14 $\times$ 14 variance-covariance matrix must be processed together. This necessitated changing the linearized form of the math model. All of the logic has been developed and most of the programming is completed at this time.

Attachments (2):
Correspondence dated November 17, 1971
and November 29, 1971

ATTACHMENT TO SECTION 2.1: (6 Pages)

U.S. DEPARTMENT OF COMMERCE
National Oceanic and Atmospheric Administration
Rockville, Md. 20852
NATIONAL OCEAN SURVEY

2574

ATTACHMENT #1

November 17, 1971

Professor Ivan I. Mueller
Dept. of Geodetic Science
The Ohio State University
164 West 19th Avenue
Columbus, Ohio 43210

Dear Professor Mueller:

Dr. Schmid has probably told you, as you had already discovered, that the type II data we sent to NASA is not what it is supposed to be. It is supposed to be Greenwich hour angle and declination referred to the mean pole of 1900-1905, the Conventional International Origin. Just a word of explanation of how this came about, then I will give you the data and formulas needed to transform the data you now have into GHA and declination and also suggest an alternative.

When the former Coast and Geodetic Survey started experimenting with satellite triangulation in the early 1960's the first observations were made from a small triangle at Aberdeen Proving Grounds in Aberdeen, Maryland. The cartesian coordinate system established for triangulation had as its origin a point on the ellipsoid directly beneath one of the camera stations. The system was left-handed with the Z axis directed positive toward the zenith, the X axis positive toward north, and the Y axis positive toward east. The geodetic ellipsoidal coordinates of the stations were referred to the 1927 North American Datum, Clarke 1866 ellipsoid.

As the experiments grew from small to large triangles, and even when projects continental and worldwide in scope were undertaken, the coordinate systems remained the same.

In recent years the entire data reduction system was modified extensively and it was decided to reprocess all of the data. It was also decided that a new reference ellipsoid, viz. Navy 8-D, would be adopted and that an ellipsoid-centered cartesian coordinate system would henceforth be used.

At the time I wrote the programs to transform our data into type I and type II data for the NASA library, they were designed to be used exclusively with the reprocessed data, i.e. an ellipsoid-centered system. Having no reprocessed data available at the time, the programs were tested with fictitious data and functioned properly.

As it turned out, for whatever reasons, the reprocessed data were referred to the Navy 8-D ellipsoid, as planned, but the cartesian coordinates continued to have as their origin the point at Aberdeen and an orientation similar to the earlier one (X north, etc.). Consequently, when these data were used to generate type II data for NASA, the quantities which we called GHA and declination were actually, or could be thought of as, an azimuth from north and ~~air~~ elevation angle referred to the cartesian origin, the point an at Aberdeen.

In order to salvage the existing erroneous type II data, one would have to refer the satellite directions to an ellipsoid-centered system by means of rotations and recompute the GHA-declination components. Details of this procedure, together with the Navy 8-D coordinates of the Aberdeen origin point are given in the enclosure for your inspection.

Since your phone call of last Friday, I have taken steps to correct the error. The program for computing type II data has been modified to compute the proper quantities. All type II data previously sent to NASA will be recalled and will be replaced by new data.

The alternative I mentioned earlier is for you to wait for correct data, rather than salvage the data you now have. Since all of the previous input to the type II data program has been recorded on magnetic tape, the correction process should be completed with in a few weeks, at most.

One other item which may also concern you has to do with type I data. While there is nothing wrong, that we know of, with the data themselves, the portion of the associated technical report describing the computation of GHA and declination from these data is, for the given reasons, also incorrect.

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A revised report will be sent to NASA with a request that old copies be destroyed and users notified accordingly.

Sincerely,

R. Hanson

Robert H. Hanson
Geodetic Research and
Development Laboratory

Enclosure

Inclosure

17 November 1971
R. Hanson

Procedures for correcting type II data

Symbols:

A — Azimuth measured clockwise positive from north at Aberdeen (incorrectly identified as Greenwich Hour Angle in type II data).

E — Elevation angle at Aberdeen (incorrectly identified as Declination in type II data).

$[\sigma^2_{AE}]$ — Covariance matrix associated with A, E above (incorrectly identified as covariance matrix for Greenwich Hour Angle – Declination).

H — Correct GHA

δ — " Declination

$[\sigma^2_{H\delta}]$ — " covariance matrix for H, δ

ϕ_0 — Latitude of origin point at Aberdeen (Navy 8-D)

λ_0 — Longitude " " " " " "

$\phi_0 = 39^\circ\ 28'\ 19.''3861$ N

$\lambda_0 = 76^\circ\ 4'\ 15.''1266$ W

1. Compute components of unit vector from A, E

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} \cos E \cos A \\ \cos E \sin A \\ \sin E \end{bmatrix} \qquad (1)$$

2. Transform X, Y, Z to ellipsoid centered system U, V, W

$$\begin{bmatrix} U \\ V \\ W \end{bmatrix} = \left[R_{\phi_0 \lambda_0}\right] \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} \cos\lambda_0 & -\sin\lambda_0 & 0 \\ \sin\lambda_0 & \cos\lambda_0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \sin\phi_0 & 0 & \cos\phi_0 \\ 0 & 1 & 0 \\ -\cos\phi_0 & 0 & \sin\phi_0 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

$$= \begin{bmatrix} -\cos\lambda_0 \sin\phi_0 & \sin\lambda_0 & \cos\lambda_0 \cos\phi_0 \\ -\sin\lambda_0 \sin\phi_0 & -\cos\lambda_0 & \sin\lambda_0 \cos\phi_0 \\ \cos\phi_0 & 0 & \sin\phi_0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \qquad (2)$$

3. Compute H, δ

$$H = \tan^{-1}\left[\frac{V}{U}\right] \quad ; \quad \text{determine quadrant}$$
$$\delta = \sin^{-1}[W] \qquad (3)$$

4. Compute covariance matrix for H, δ

$$\left[\frac{\partial(XYZ)}{\partial(AE)}\right] = \begin{bmatrix} \frac{\partial X}{\partial A} & \frac{\partial X}{\partial E} \\ \frac{\partial Y}{\partial A} & \frac{\partial Y}{\partial E} \\ \frac{\partial Z}{\partial A} & \frac{\partial Z}{\partial E} \end{bmatrix} = \begin{bmatrix} -\cos E \sin A & -\sin E \cos A \\ \cos E \cos A & -\sin E \sin A \\ 0 & \cos E \end{bmatrix} \qquad (4)$$

$$\left[\frac{\partial(UVW)}{\partial(XYZ)}\right] = \begin{bmatrix} \frac{\partial U}{\partial X} & \frac{\partial U}{\partial Y} & \frac{\partial U}{\partial Z} \\ \frac{\partial V}{\partial X} & \frac{\partial V}{\partial Y} & \frac{\partial V}{\partial Z} \\ \frac{\partial W}{\partial X} & \frac{\partial W}{\partial Y} & \frac{\partial W}{\partial Z} \end{bmatrix} = \left[R_{\phi_0\lambda_0}\right] \tag{5}$$

$$\left[\frac{\partial(H\delta)}{\partial(UVW)}\right] = \begin{bmatrix} \frac{\partial H}{\partial U} & \frac{\partial H}{\partial V} & \frac{\partial H}{\partial W} \\ \frac{\partial \delta}{\partial U} & \frac{\partial \delta}{\partial V} & \frac{\partial \delta}{\partial W} \end{bmatrix} = \frac{1}{\cos\delta}\begin{bmatrix} \frac{-V}{\cos\delta} & \frac{U}{\cos\delta} & 0 \\ -UW & -VW & \cos^2\delta \end{bmatrix} \tag{6}$$

$$\left[\frac{\partial(H\delta)}{\partial(AE)}\right] = \begin{bmatrix} \frac{\partial H}{\partial A} & \frac{\partial H}{\partial E} \\ \frac{\partial \delta}{\partial A} & \frac{\partial \delta}{\partial E} \end{bmatrix} = \left[\frac{\partial(H\delta)}{\partial(UVW)}\right]\left[\frac{\partial(UVW)}{\partial(XYZ)}\right]\left[\frac{\partial(XYZ)}{\partial(AE)}\right] \tag{7}$$

$$\underset{2n \times 2n}{\left[\sigma^2_{H\delta}\right]} = \underbrace{\left[\frac{\partial(H\delta)}{\partial(AE)}\right]}_{\text{2x2 blocks along diagonal}}\underbrace{\left[\sigma^2_{AE}\right]}_{\text{full 2nx2n matrix}}\left[\frac{\partial(H\delta)}{\partial(AE)}\right]^T \tag{8}$$

n = No. of directions

U.S. DEPARTMENT OF COMMERCE
National Oceanic and Atmospheric Administration
Rockville, Md. 20852
NATIONAL OCEAN SURVEY

ATTACHMENT #2

November 29, 1971

Mr. Jerome D. Rosenberg
Deputy Director, Communications Programs
Office of Space Science and Applications
National Aeronautics and Space Administration
Washington, D. C. 20546

Dear Mr. Rosenberg:

Attached is a revised version of the technical report for Contract W-13,321, dated January 19, 1971. Please note that only those pages containing changes have been marked "revised."

Sincerely,

Hellmut Schmid

Hellmut H. Schmid
Director, Geodetic Research
and Development Laboratory

Enclosure

cc: ✓Prof. Ivan I. Mueller,
Ohio State University

July 1, 1971
(Revised Nov. 18, 1971)

Technical Report - Contract W-13, 321

In accordance with contract W-13,321, dated January 19, 1971, we are submitting the following report describing the contents of the magnetic tapes being supplied to the NASA Space Science Data Center, containing data from the BC-4 worldwide PAGEOS network.

Section I

Data Format of the Magnetic Tapes

The data is furnished on 1/2" BCD (even parity) 7-track tapes at 556 bits/inch. Each record is 80 characters long. The first file on each tape contains an index to the contents of the tape, listing event numbers and the stations associated with each. The remaining files each contain data from one event.

A detailed description of the data formats follows: FORTRAN field specifications are given for numeric data fields. All unspecified fields are alphanumeric.

1. Type 1 - Partially Reduced Observations

   a. For each event:

Event record	Col.	1 - 6		Event number
		7 - 23		Date of observation
		24 - 47		Satellite name
		48 - 53	F6.2	Satellite radius (m)
		54 -	I1	No. of stations viewing event
		55 - 61	F7.4	x-angle } coordinates of instantaneous pole relative to CIO (sec. of arc)
		62 - 68	F7.4	y-angle } coordinates of instantaneous pole relative to CIO (sec. of arc)

b. For each plate:

Rec. 1 Col.	1 - 6		Station number
	7 - 30		Station name
	31 - 33	I3	Station latitude ϕ^{o}
	35 - 36	I2	Station latitude ϕ'
	37 - 44	F8.4	Station latitude ϕ''
	45 - 48	I4	Station longitude λ^{o}
	50 - 51	I2	Station longitude λ'
	52 - 59	F8.4	Station longitude λ''
Rec. 2 Col.	1 - 14	E14.7	Station temperature (°C)
	15 - 28	E14.7	Station pressure (mm of Hg)
	29 - 32		Plate number
	33 - 36	I4	Number of satellite images
Rec. 3 Col.	1 - 48	3E16.9	Local camera orientation angles (radians)
	49 - 64	E16.9	Angle of nonperpendicularity of plate coordinates (radians)
	65 - 80	E16.9	x coordinate of principal point (m)
Rec. 4 Col.	1 - 16	E16.9	y coordinate of principal point (m)
	17 - 48	2E16.9	x and y coordinate scalars (m)
	49 - 80	2E16.9	x and y coordinates (relative to principal point) of zero distortion (m)
Rec. 5 Col.	1 - 48	3E16.9	3 coefficients of radial distortion
	49 - 80	2E16.9	2 coefficients of decentering distortion
Rec. 6 Col.	1 - 16	E16.9	Angle of axis of lens distortion (radians
Rec. 7 thru 40 Col.	1 - 80	4E20.13	Upper triangular part of 16x16 covariance matrix associated with camera orientation parameters (by rows) (136 terms total)
Rec. 41 Col.	1 - 48	3E16.9	Transformed camera orientation angles (radians)

c. For each satellite image

Image record	Col. 1 - 8	I8	Image number	
	9 - 36	2E14.7	Adjusted x,y plate coordinates (m)	
	38 - 39	I2	LST	hr
	41 - 42	I2	of	min
	43 - 52	F10.6	observation	sec
	54 - 55	I2	UT1	hr
	57 - 58	I2	of	min
	59 - 68	F10.6	observation	sec

2. Type 2 - Fictitious Satellite Directions

a. For each event

Event record	Col. 1 - 6		Event number
	7 -	I1	Number of stations
	8 - 9	I2	Total number of fictitious

b. For each plate

Rec. 1	Col. 1 - 6		Station number
	7 - 30		Station name
	31 - 34		Plate number
	35 - 36	I2	Number of usable fictitious images
Matrix records	Col. 1 - 80	4E20.13	Upper triangular part of covarianc matrix associated with fictitious directions (radians squared).

(Note: No. of terms (N) in the upper triangular part of the matrix depends on total no. (n) of fictitious points: N=n(2n+1))

c. For each fictitious point

Record	Col. 1 - 2	I2	Point number
	3 - 18	E16.9	Greenwich hour angle (radians)
	19 - 34	E16.9	Declination (radians)

Section II.

Further Reduction of Partially Reduced Satellite Image Plate Coordinates

This section of the report outlines the computational steps required to correct the partially reduced satellite image plate coordinates for aberrations peculiar to the particular comparator and camera with which they are associated, and to propagate the uncertainties of camera orientation into fully correlated image coordinate covariance matrices. Formulas for transforming the corrected satellite image plate coordinates into directional coordinates and a discussion of external aberrations will also be given.

1. Symbols Used for Given Data:

ϕ --- Nominal latitude of camera station.

x_a, y_a --- Coordinates (angles) of earth's instantaneous pole relative to the CIO mean pole of 1900-1905.

x, y --- Partially reduced satellite image plate coordinates.

$\alpha_L, \omega_L, \kappa_L$ --- Camera orientation angles relative to the local station coordinate system.

$\alpha_T, \omega_T, \kappa_T$ --- Camera orientation angles relative to the coordinate system used in satellite triangulation adjustment (transformed from local system).

ϵ --- Angle of non-perpendicularity of the plate coordinate system.

x_p, y_p --- Plate coordinates of the camera principal point.

C_x, C_y --- X and Y plate coordinate scalars.

Δ_x, Δ_y --- Plate coordinates of the point having zero distortion (relative to the principal point).

K_1, K_2, K_3 --- Coefficients of radial lens distortion polynomial.

K_4, K_5 --- Coefficients of a function describing decentering lens distortion.

ϕ_t --- An angle which relates the axis of lens distortion symmetry to the plate coordinate Y axis.

$\left[\sigma_o^2\right]$ --- Covariance matrix associated with camera orientation parameters.

2. Satellite Image Coordinate Corrections:

In order to be correct, the following computations must be done in the specified order:

a. Correct for non-perpendicularity of axes:

$$x' = x + y\epsilon \; ; \qquad \epsilon = \text{angle in radians} \tag{1}$$

$$y' = y \tag{2}$$

b. Correct for radial and decentering lens distortions:

$$x^* = x' \tag{3}$$

$$y^* = y' \tag{4}$$

$$D_x = x^* - x_p - \Delta_x \tag{5}$$

$$D_y = y^* - y_p - \Delta_y \tag{6}$$

$$d^2 = D_x^2 + D_y^2 \tag{7}$$

$$\frac{\Delta_R}{d} = K_1 d^2 + K_2 d^4 + K_3 d^6 \tag{8}$$

$$\frac{\Delta_D}{d^2} = K_4 + K_5 d^2 \tag{9}$$

$$A = D_x \cos\phi_t + D_y \sin\phi_t \tag{10}$$

$$x'' = x' - \frac{\Delta_R}{d} D_x - \frac{\Delta_D}{d^2} (2D_x A + d^2 \cos\phi_t) \tag{11}$$

$$y'' = y' - \frac{\Delta_R}{d} D_y - \frac{\Delta_D}{d^2} (2D_y A + d^2 \sin\phi_t) \tag{12}$$

Because the distortion formulas are functions of undistorted distances, and because the given coordinates are distorted, it is necessary to iterate as follows:

(1) After computing (5) thru (12) initially, set

$x^* = x''$

$y^* = y''$

and repeat (5) thru (12).

(2) Compare $|\Delta_{R_n} - \Delta_{R_{n-1}}|$ against a pre-set tolerance (0.1 micrometers is typical) and compare $|\Delta_{D_n} - \Delta_{D_{n-1}}|$ against a tolerance.

If both differences are less than or equal to the tolerances, continue with next set of corrections. Otherwise, set x^* and y^* equal to the latest x'' and y'' and repeat computations (5) thru (12).

c. Translate coordinates to principal point

$$x''' = x'' - x_p \tag{13}$$

$$y''' = y'' - y_p \tag{14}$$

d. Correct for scale differences

$$x'''' = x''' \tag{15}$$

$$y'''' = \frac{C_x}{C_y} y''' \tag{16}$$

. At this stage, following corrections in a. thru d. above, the satellite image plate coordinates should be uncontaminated by any known, systematic errors originating from the measuring or camera orientation process. The uncertainties of these coordinates attributable solely to uncertainties in the camera orientation parameters can be expressed by the co-variance matrix discussed in the next paragraph.

3. Propagation of Camera Orientation Errors:

Henceforth, for simplicity's sake, let x'''' and y'''' be denoted by x and y, remembering that these symbols now stand for corrected satellite image plate coordinates and not, as originally, for given coordinates.

a. Direction Cosines

$$[R] = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \tag{17}$$

$$R_{11} = -\cos\alpha\cos\kappa+\sin\alpha\sin\omega\sin\kappa \tag{18}$$

$$R_{21} = -\cos\omega\sin\kappa$$

$$R_{31} = \sin\alpha\cos\kappa+\cos\alpha\sin\omega\sin\kappa$$

$$R_{12} = -\cos\alpha\sin\kappa-\sin\alpha\sin\omega\cos\kappa$$

$$R_{22} = \cos\omega\cos\kappa$$

$$R_{32} = \sin\alpha\sin\kappa-\cos\alpha\sin\omega\cos\kappa$$

$$R_{13} = \sin\alpha\cos\omega$$

$$R_{23} = \sin\omega$$

$$R_{33} = \cos\alpha\cos\omega$$

$[R_L]$ = Matrix computed with local orientation angles.

$[R_T]$ = Matrix computed with transformed orientation angles.

b. Computational Auxiliaries

(Note: directions cosines are from $[R_L]$)

$$① = \frac{x}{C_x} \tag{19}$$

$$② = \frac{y}{C_y} \tag{20}$$

$$③ = ① R_{13}-R_{11} \tag{21}$$

$$\textcircled{4} = \textcircled{1} R_{23} - R_{21} \tag{22}$$

$$\textcircled{5} = \textcircled{2} R_{13} - R_{12} \tag{23}$$

$$\textcircled{6} = \textcircled{2} R_{23} - R_{22} \tag{24}$$

$$\textcircled{7} = \textcircled{2} R_{21} - \textcircled{1} R_{22} \tag{25}$$

$$\textcircled{8} = x - \Delta_x \tag{26}$$

$$\textcircled{9} = y - \Delta_y \tag{27}$$

$$\textcircled{10} = \textcircled{8}^2 + \textcircled{9}^2 \tag{28}$$

$$\textcircled{11} = \left[2k_1 + 4k_2 \textcircled{10} + 6k_3 \textcircled{10}^2\right] \tag{29}$$

$$\textcircled{12} = \left[2k_4 + 4k_5 \textcircled{10}\right] . \quad (30$$

$$\textcircled{13} = k_1 \textcircled{10} + k_2 \textcircled{10}^2 + k_3 \textcircled{10}^3 \tag{31}$$

$$\textcircled{14} = k_4 \textcircled{10} + k_5 \textcircled{10}^2 \tag{32}$$

$$\textcircled{15} = k_4 + (\textcircled{8}^2 + 3 \textcircled{9}^2) k_5 \tag{33}$$

$$\textcircled{16} = \textcircled{9} \cos\phi_t - \textcircled{8} \sin\phi_t \tag{34}$$

$$\textcircled{17} = \textcircled{8} \cos\phi_t + \textcircled{9} \sin\phi_t \tag{35}$$

c. Partial derivatives

$$\frac{\partial x}{\partial \alpha} = -C_x(\textcircled{1}\,\textcircled{7} + \textcircled{6}) \tag{36}$$

$$\frac{\partial y}{\partial \alpha} = -C_y(\textcircled{2}\,\textcircled{7} - \textcircled{4}) \tag{37}$$

$$\frac{\partial x}{\partial \omega} = C_x\left[(1 + \textcircled{1}^2)\sin\kappa_L - \textcircled{1}\,\textcircled{2}\cos\kappa_L\right] \tag{38}$$

$$\frac{\partial y}{\partial \omega} = -C_y\left[(1 + \textcircled{2}^2)\cos\kappa_L - \textcircled{1}\,\textcircled{2}\sin\kappa_L\right] \tag{39}$$

$$\frac{\partial x}{\partial \kappa} = -C_x \textcircled{2} \tag{40}$$

$$\frac{\partial y}{\partial \kappa} = C_y \; 1 \tag{41}$$

$$\frac{\partial x}{\partial \varepsilon} = -y \tag{42}$$

$$\frac{\partial y}{\partial \varepsilon} = 0 \tag{43}$$

$$\frac{\partial x}{\partial x_p} = 1 \tag{44}$$

$$\frac{\partial y}{\partial x_p} = 0 \tag{45}$$

$$\frac{\partial x}{\partial y_p} = 0 \tag{46}$$

$$\frac{\partial y}{\partial y_p} = 1 \tag{47}$$

$$\frac{\partial x}{\partial C_x} = 1 \tag{48}$$

$$\frac{\partial y}{\partial C_x} = 0 \tag{49}$$

$$\frac{\partial x}{\partial C_y} = 0 \tag{50}$$

$$\frac{\partial y}{\partial C_y} = 2 \tag{51}$$

$$\begin{aligned}\frac{\partial x}{\partial \Delta_x} = &- 13 - 8^2 \; 11 - 8 \; 12 \cos\phi_t \\ &-4 \; 8 \left[k_4 + (2 \; 8^2 + 9^2) k_5\right] \cos\phi_t \\ &-2 \; 9 \; 15 \sin\phi_t\end{aligned} \tag{52}$$

$$\begin{aligned}\frac{\partial y}{\partial \Delta_x} = &- 8 \; 9 \; 11 - 8 \; 12 \sin\phi_t \\ &-2 \; 9 \left\{\left[k_4 + (3 \; 8^2 + 9^2) k_5\right] \cos\phi_t \right. \\ &\left. +2 \; 9 \; k_5 \sin\phi_t\right\}\end{aligned} \tag{53}$$

$$\begin{aligned}\frac{\partial x}{\partial \Delta_y} = &- 8 \; 9 \; 11 - 9 \; 12 \cos\phi_t - 2 \; 8 \, (15 \sin\phi_t \\ &+2 \; 8 \; 9 \; k_5 \cos\phi_t)\end{aligned} \tag{54}$$

$$\frac{\partial y}{\partial \Delta_y} = -⑬ - ⑨^2 ⑪ - ⑨ ⑫ \sin\phi_t - 2⑧ ⑮ \cos\phi_t$$
$$-4⑨\left[k_4 + (⑧^2 + 2⑨^2)k_5\right]\sin\phi_t \quad (55)$$

$$\frac{\partial x}{\partial k_1} = ⑧ ⑩ \quad (56)$$

$$\frac{\partial y}{\partial k_1} = ⑨ ⑩ \quad (57)$$

$$\frac{\partial x}{\partial k_2} = ⑧ ⑩^2 \quad (58)$$

$$\frac{\partial y}{\partial k_2} = ⑨ ⑩^2 \quad (59)$$

$$\frac{\partial x}{\partial k_3} = ⑧ ⑩^3 \quad (60)$$

$$\frac{\partial y}{\partial k_3} = ⑨ ⑩^3 \quad (61)$$

$$\frac{\partial x}{\partial k_4} = ⑩ \cos\phi_t + 2⑧ ⑰ \quad (62)$$

$$\frac{\partial y}{\partial k_4} = ⑩ \sin\phi_t + 2⑨ ⑰ \quad (63)$$

$$\frac{\partial x}{\partial k_5} = ⑩ \frac{\partial x}{\partial k_4} \quad (64)$$

$$\frac{\partial y}{\partial k_5} = ⑩ \frac{\partial y}{\partial k_4} \quad (65)$$

$$\frac{\partial x}{\partial \phi_t} = ⑭\left[\frac{2⑧⑯}{⑩} - \sin\phi_t\right] \quad (66)$$

$$\frac{\partial y}{\partial \phi_t} = ⑭\left[\frac{2⑨⑯}{⑩} + \cos\phi_t\right] \quad (67)$$

d. Plate coordinate covariance matrix

$$\left[\frac{\partial xy}{\partial O}\right]=\begin{bmatrix} \frac{\partial x}{\partial \alpha} & \frac{\partial x}{\partial \omega} & \frac{\partial x}{\partial \kappa} & \frac{\partial x}{\partial \epsilon} & \frac{\partial x}{\partial x_p} & \frac{\partial x}{\partial y_p} & \frac{\partial x}{\partial C_x} & \frac{\partial x}{\partial C_y} & \frac{\partial x}{\partial \Delta_x} & \frac{\partial x}{\partial \Delta_y} & \frac{\partial x}{\partial k_1} & \frac{\partial x}{\partial k_2} & \frac{\partial x}{\partial k_3} & \frac{\partial x}{\partial k_4} & \frac{\partial x}{\partial k_5} & \frac{\partial x}{\partial \phi_t} \\ \frac{\partial y}{\partial \alpha} & \frac{\partial y}{\partial \omega} & \frac{\partial y}{\partial \kappa} & \frac{\partial y}{\partial \epsilon} & \frac{\partial y}{\partial x_p} & \frac{\partial y}{\partial y_p} & \frac{\partial y}{\partial C_x} & \frac{\partial y}{\partial C_y} & \frac{\partial y}{\partial \Delta_x} & \frac{\partial y}{\partial \Delta_y} & \frac{\partial y}{\partial k_1} & \frac{\partial y}{\partial k_2} & \frac{\partial y}{\partial k_3} & \frac{\partial y}{\partial k_4} & \frac{\partial y}{\partial k_5} & \frac{\partial y}{\partial \phi_t} \end{bmatrix} \tag{68}$$

$$\underset{(2\times 2)}{\left[\sigma^2_{xy_o}\right]}=\underset{(2\times 16)}{\left[\frac{\partial xy}{\partial O}\right]}\underset{(16\times 16)}{\left[\ \sigma_o{}^2\ \right]}\underset{(16\times 2)}{\left[\frac{\partial xy}{\partial O}\right]^T} \quad \text{for a single satellite image} \tag{69}$$

or

$$\underset{(2n\times 2n)}{\left[\sigma^2_{xy_o}\right]}=\underset{(2n\times 16)}{\left[\frac{\partial xy}{\partial O}\right]}\underset{(16\times 16)}{\left[\ \sigma_o{}^2\ \right]}\underset{(16\times 2n)}{\left[\frac{\partial xy}{\partial O}\right]^T} \quad \text{for n satellite images} \quad (n \leqq 8) \tag{70}$$

4. Satellite Directions and External Aberrations:

a. Azimuth from south (A) and zenith distance (Z_R)

$$\begin{bmatrix} U \\ V \\ W \end{bmatrix} = \left[R_L\right]\begin{bmatrix} x \\ y \\ C_x \end{bmatrix} \tag{71}$$

$$\text{if } U = 0 \text{ and } V \leqq 0,\ A = \frac{\pi}{2} \tag{72}$$

$$\text{if } U = 0 \text{ and } V > 0,\ A = \frac{3\pi}{2}$$

$$K = \tan^{-1}\left(\frac{V}{U}\right)$$

$$\text{if } U < 0 \text{ and } V > 0,\ A = K + 2\pi$$

$$\text{if } U < 0 \text{ and } V \leqq 0,\ A = K$$

$$\text{if } U > 0,\ A = K + \pi$$

if $W = 0,\ Z_R = \frac{\pi}{2}$ (73)

if $W \neq 0,\ Z_R = \tan^{-1}\left[\frac{(U^2+V^2)^{\frac{1}{2}}}{W}\right]$

b. Atmospheric refraction

It is customary at this stage to correct the zenith distance (Z_R) for atmospheric refraction before proceeding to the next step.

c. Local hour angle (H) and declination (δ) relative to instantaneous pole

$Z = Z_R$ corrected for atmospheric refraction (74)

$$\delta = \sin^{-1}(\cos Z \sin\phi - \sin Z \cos A \cos\phi) \quad (75)$$

$$Y = \sin Z \sin A \quad (76)$$

$$X = \cos Z \cos\phi + \sin Z \cos A \sin\phi \quad (77)$$

if $X = 0$ and $Y \geqq 0,\ H = \frac{\pi}{2}$ (78)

if $X = 0$ and $Y < 0,\ H = \frac{3\pi}{2}$

$K = \tan^{-1}\left(\frac{Y}{X}\right)$

if $X > 0$ and $Y < 0,\ H = K+2\pi$

if $X > 0$ and $Y > 0,\ H = K$

if $X < 0,\quad H = K+\pi$

d. Phase angle

Corrections to H and δ for phase angle, the displacement of the sun's reflected image from the satellite center, are most conveniently made at this stage.

e. Right ascension

The topocentric right ascension at the time of observation can be computed as: local sidereal time of observation minus the local hour angle (H).

5. Plate Coordinates from Corrected Directions:

$$\cos Z' = \sin\phi\sin\delta+\cos\phi\cos\delta\cos H \tag{79}$$

$$\xi = \frac{\cos\phi\sin\delta-\sin\phi\cos\delta\cos H}{\cos Z'} \tag{80}$$

$$\eta = \frac{-\cos\delta\sin H}{\cos Z'} \tag{81}$$

$$\begin{bmatrix} m \\ n \\ q \end{bmatrix} = \begin{bmatrix} R_L \end{bmatrix}^T \begin{bmatrix} \xi \\ \eta \\ 1 \end{bmatrix} \tag{82}$$

$$x' = C_x \frac{m}{q} \tag{83}$$

$$y' = C_x \frac{n}{q} \tag{84}$$

6. Greenwich Hour Angle (H') and Declination (δ') Relative to CIO Mean Pole (1900-1905):

$$\begin{bmatrix} R_{\varphi_o\lambda_o} \end{bmatrix} = \begin{bmatrix} -\cos\lambda_o\sin\varphi_o & \sin\lambda_o & \cos\lambda_o\cos\varphi_o \\ -\sin\lambda_o\sin\varphi_o & -\cos\lambda_o & \sin\lambda_o\cos\varphi_o \\ \cos\varphi_o & 0 & \sin\varphi_o \end{bmatrix} \tag{85}$$

$\varphi_o = 39^\circ 28' 19''.3861$ N; $\lambda_o = 76^\circ 4' 15''.1266$ W

$$\begin{bmatrix} U^* \\ V^* \\ W^* \end{bmatrix} = \begin{bmatrix} R_{\varphi_o\lambda_o} \end{bmatrix} \begin{bmatrix} R_T \end{bmatrix} \begin{bmatrix} x' \\ y' \\ C_x \end{bmatrix} \tag{86}$$

Repeat computations (72) and (73) using U*,V* and W* to obtain A* and Z*, then:

$$H' = A^* + \pi \quad (\text{if } H' \geq 2\pi, \text{ subtract } 2\pi) \tag{87}$$

$$\delta' = \frac{\pi}{2} - Z^* \tag{88}$$

2.2 Investigations Related to the Problem of Improving Existing Triangulation Systems by Means of Satellite Super-Control Points

This investigation was completed during this reporting period, the details of which will be published separately under Reports of the Department of Geodetic Science.

The objective of this investigation was to answer the question: Whether any significant increment to accuracy could be transferred from a super-control net (continental satellite net or super-transcontinental traverse) to the basic geodetic net (First-order triangulation). This objective was achieved by evaluating the position of accuracy improvement for a triangulation station, which is near the middle of the investigated geodetic triangulation net.

For the purpose of the present investigation, the triangulation of the western-half of the United States has been considered, as this is more accurate than that of the eastern-half of the United States [Simmons, 1950, p. 54]. The investigation is done on the chain from Moses Lake, Washington to Chandler, Minnesota (Figure 1), as these two stations are also common on both the continental satellite net (CSN) and the super-transcontinental traverse (STT). The data was supplied by the Triangulation Branch of Geodesy Division, and the Geodetic Research and Development Laboratory, both of the National Oceanic and Atmospheric Administration, Washington.

The geodetic triangulation net is adjusted as an independent or free net, as it is not connected with other nets. For its unambiguous determination, besides the observed data which includes directions, bases (to provide the scale) and astronomical observations, i.e., longitude and azimuth (to provide orientation of the triangulation net upon a mathematical surface, i.e., ellipsoid), one fixed station is required to serve as origin [Gotthardt, 1968, p. 167]. Moses Lake station is kept as origin with its coordinates obtained from satellite triangulation results; these coordinates have been assumed to be the best known coordinates.

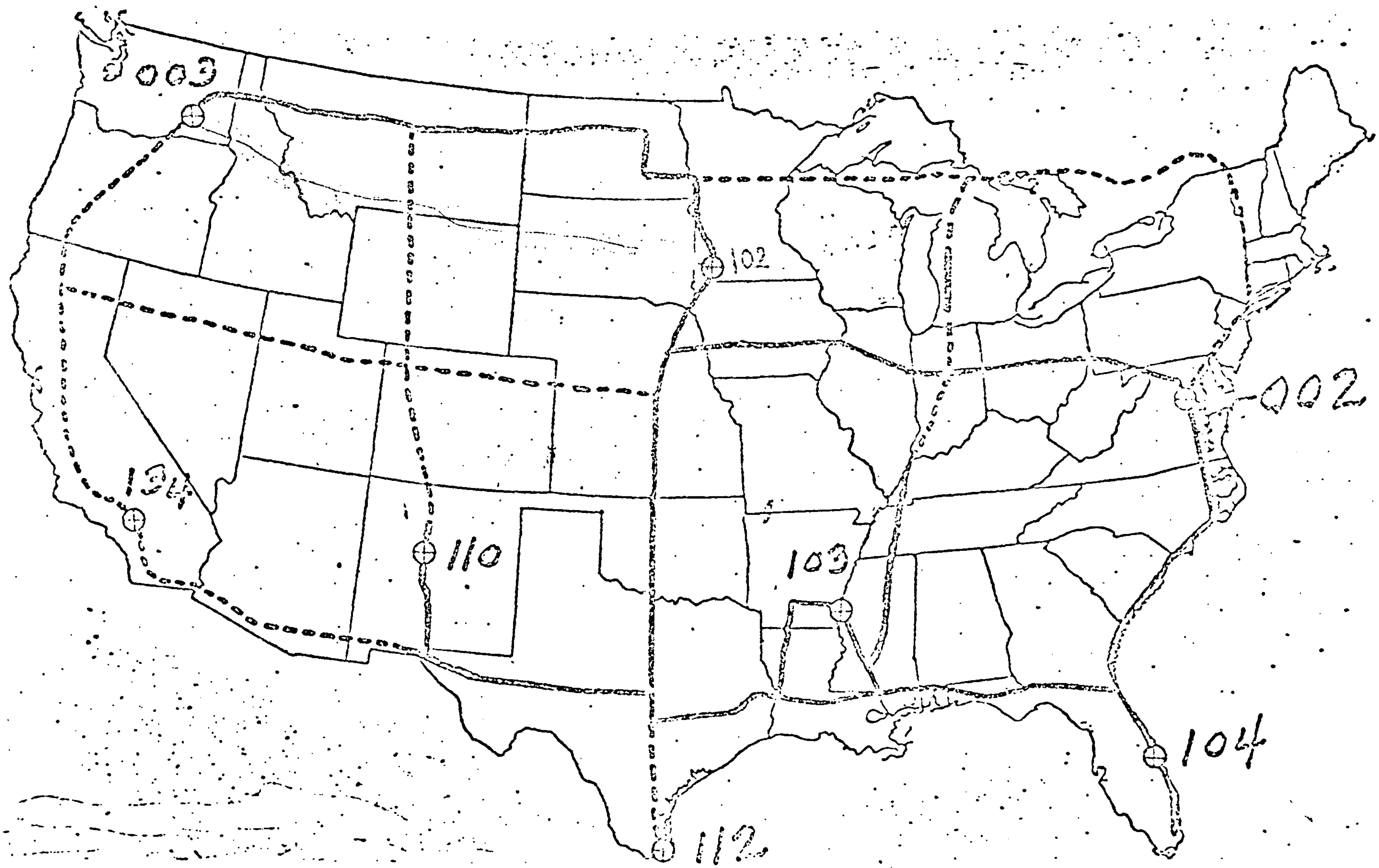

Figure 1. General Location of Geodetic Triangulation Chain from Moses Lake (003) to Chandler (102).

Combining the free triangulation net with super-control net of zero' order, i. e., continental satellite net and/or super transcontinental traverse means constraining the scale and/or orientation of the triangulation net. The effect of this combination is comparable with "tennis racket and string effect", where the rigid outer racket frame (super-control) constrains the loose strings (triangulation net). If the strings are already constrained, there will be no "visible" effect of the additional constrain from the rigid outer frame. This is also the purpose of this investigation, i. e., to evaluate whether the existing geodetic triangulation is sufficiently "constrained" or needs to be constrained by additional super-control net. For the present investigation a triangulation station Chandler, which is common to the three networks, provides constraint.

Geodetic triangulation net can be combined with the super-control net in either of the two ways:

(1) By using the actual data, i. e., by using the actual coordinates with their standard errors of Chandler as obtained from CSN and STT with the geodetic triangulation; or,

(2) By adding a weight constraint to Chandler with its coordinates from the geodetic triangulation.

For this investigation, the first way could not be used, as the super-control net coordinates of Chandler station are not compatible with those obtained from geodetic triangulation. As such, the second way has been preferred by using the actual preliminary accuracy estimates for Chandler, which are 1 part in 385,000 and 1 part in 3 million, as obtained from CSN and STT, respectively. Further investigations are made by using hypothetical standard positional error accuracy estimates of Chandler station, which are 1:400,000; 1:500,000; 1:600,000; 1:700,000; 1:1M; 1:1,5M. These accuracy estimates are within the actual preliminary accuracy estimates of super-control nets. Thus, using these various accuracies of super-control net, a feeling for the accuracy limit of super-control net, which would be necessary to improve the investigated geodetic triangulation, can be obtained.

The Conjugate Gradient Method (Cg Method) is used for adjustment, which uses the original homogenized observation equations. Cg-Method has been programmed in such a way so as to use varying data with only change in the dimension statement.

The results of the investigation are given in Tables 1 and 2, wherein the improvement of the particular geodetic triangulation by super-control net is visible only when its accuracy is at least 1 part in 500,000. The positional improvement of Wyola (95), which is in the middle of the triangulation chain, using various station constraints for Chandler (3) relative to free net adjustment are shown in Figure 2. As the preliminary accuracy of continental satellite net is lower than 1 part in 500,000, this cannot be useful as a "constraint" to the geodetic triangulation net. On the other hand, the high accuracy of super-transcontinental traverse, which is one part in 3 million, makes it most suitable as a "constraint" to the geodetic triangulation net.

Worth mentioning is that the longitude terms, which are Q_{yy} and $\sigma_y{}^2$ in Table 1, remain practically uneffected during the entire investigation. This could be explained by the fact that station Wyola is very close to Laplace stations, which control the azimuth error accumulation, thus effecting the longitude error [Bomford, 1962, pp. 90, 519]. Hence, due to closeness of Laplace stations, the longitude terms remain practically uneffected.

The super-control net, i.e., continental satellite net or super-transcontinental traverse, can provide a useful constraint to the investigated geodetic triangulation net, and thus can improve it only when the accuracy of super-control net is at least 1 part in 500,000 in this case, this corresponds to $\pm$ 3.7 m standard position error to the station Chandler. The preliminary accuracy of super-transcontinental traverse is already better than this limiting accuracy of 1 part in 500,000. The preliminary accuracy of continental satellite net is, however, lower than the limiting accuracy of 1 part in 500,000; the preliminary standard positional error for Chandler as obtained from continental satellite net corresponds to $\pm$4.8 m, i.e., 1 part is 385,000. The future will show whether

Table 1

Experiment Number	Accuracy 1 in	$\hat{m}_o$	WYOLA (95)				Remarks
			Q_{xx}	Q_{yy}	m_x^2	m_y^2	
1		2.42	6.0	0.5	35.2	2.9	Free Net
2	300,000	2.41	6.7	0.5	38.9	2.9	
3	400,000	2.41	5.9	0.5	34.3	2.9	
4	500,000	2.41	4.1	0.5	23.8	2.9	
5	600,000	2.41	4.1	0.5	23.8	2.9	
6	700,000	2.41	4.1	0.5	23.8	2.9	
7	1,000,000	2.41	3.7	0.5	21.5	2.9	
8	1,500,000	2.41	3.2	0.5	18.6	2.9	
9	3,000,000	2.41	2.1	0.5	12.2	2.9	

Q_{xx}, Q_{yy} and m_x^2, m_y^2 are given in 10^{-4} seconds^2.

Table 2

Experiment Number	Accuracy 1 in	WYOLA (95)				
					Positional Improvement Relative to Experiment 1	
		m_x	m_y	m_p	Meters	%
1	Free Net	1.83	0.37	1.9		
2	300,000	1.93	0.37	2.0	-0.1	- 5
3	400,000	1.81	0.37	1.8	0.1	5
4	500,000	1.51	0.37	1.5	0.4	21
5	600,000	1.51	0.37	1.5	0.4	21
6	700,000	1.51	0.37	1.5	0.4	21
7	1,000,000	1.43	0.37	1.5	0.4	21
8	1,500,000	1.33	0.37	1.4	0.5	26
9	3,000,000	1.08	0.37	1.1	0.8	42

Standard Errors of Unknowns (m_x, m_y) and Standard Positional Error (m_p) are given in meters.

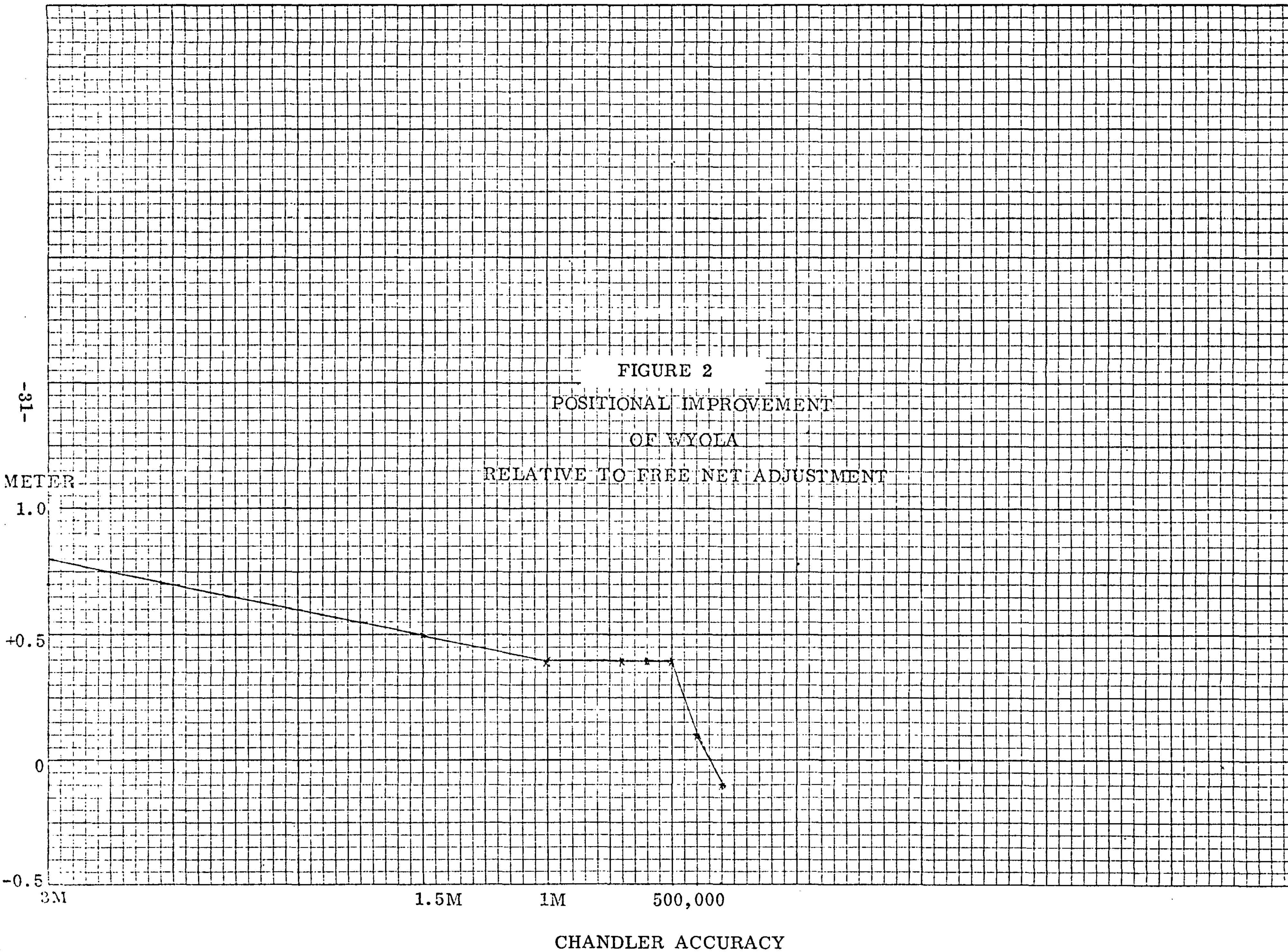

FIGURE 2
POSITIONAL IMPROVEMENT
OF WYOLA
RELATIVE TO FREE NET ADJUSTMENT

this limiting accuracy could be achieved by continental satellite net, especially because numerous spatial triangulation of CSN have produced accuracies within the range of 1 part in 400,000 and 1 part in 700,000 [Schmid, 1965, p. 22]. Schmid [1970, p. 23-24] indicates that continental satellite net will fall short of an optimum solution with respect to both its coverage and its accuracy. The non-achievement of the anticipated accuracy may be due to (1) the earlier observations made with the 300 mm lens, which have a slight disadvantage when compared to the presently available 450 mm lens optimized for satellite triangulation, (2) the lack of knowledge of the minute orientation changes of BC-4 system in the earlier phase, which occur occasionally between the period separating pre- and post-star calibrations, and (3) the absence of an optimal target after the demise of Echo I and II satellites, only the PAGEOS satellite is available, which, with an average slant distance of six million meters yields results degraded by a factor of three, compared with results, obtainable with a balloon satellite at the optimum height of 1500 km above the earth. Thus, the three-dimensional positions of CSN-stations will probably be determined to no better than ±4 meters in all components which does not seem to be good enough at least for this particular triangulation chain. It might be useful to have a "block constrain" instead of "chain constrain", that is, to use four well-separated satellite stations 003, 102, 112, and 134 (Figure 1) over a very large area, thus constraining the triangulation of the western-half of the United States instead of one triangulation chain ("chain constrain") between stations 003 and 102.

Super-transcontinental traverse can provide a better constraint, if more than two of its stations are common to the stations of geodetic triangulation net. Also, a "block constrain" as explained above, might be more useful instead of a "chain constrain".

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2.3 Scaling The SAO-69 Geometric Solution With C-Band Radar Data (Solution SC 11)

SC 11 is an adjustment of the SAO optical observations combined with pseudo observations of chord distances derived from the adjustment of the C-Band network. Weighted height constraints (mean sea level + SAO 69 undulations) were impared at most stations as a means of introducing valuable additional information.

14,356 optical simultaneous observations from 28 stations were received from the SAO. The number and distribution of these observations are shown in Figures 1 and 2.

The C-Band data consisted of the results published by NASA-Wallops Island [1]. Upon request, they also kindly supplied us with the correlation matrix of their adjustment from which it was easy to compute the variance-covariance matrix. This adjustment was developed from about 2,000,000 range observations in which 466 tracks of GEOS-II were observed by twenty-one C-Band radars.

Only four C-Band stations were near enough to the SAO stations so that the relative positions could be reliably determined through first order triangulation. To account for any error introduced through the triangulation, these ties were accounted for by imposing weighted relative position constraints on the adjustment.

As the geometry of the network did not always guarantee a unique solution, it was also necessary to impose a few weighted direction constraints derived from the SAO 69 adjustment [2]. These direction constraints imposed were between

Addes Ababa and Shiraz

Addes Ababa and Natal

Natal and Pretoria

Natal and Villa Dolores

Natal and Arequipa

Natal and Curacao

Organ Pass and Rosamund

The above data and constraints were used as input in the least squares adjustment program. Several adjustments were attempted, but the final one chosen, the SC 11, used only the two C-Band chord lengths (Kauai to Merritt Island and Merritt Island to Pretoria). In this adjustment two iterations were carried out from which a new set of coordinates for the SAO Network, and the four C-Band radar sites were obtained.

The results of our adjustment are given in Table 1. In Table 2, the actual differences from the SAO 69 solution are given along with the standard deviations of the corresponding coordinates. For comparison, also included are the standard deviations obtained by the SAO. Entries enclosed in a box are statistically significant at the 3σ level. The remaining differences contain zero within their confidence intervals and are assumed insignificant. In those cases where the difference is significant, the SAO and our standard deviations are of the same order of magnitude.

Tables 3 and 4 give chord-distance comparison with other adjustments on these stations. The SC 11 solution is compared with both SAO 69 and with GSFC 1971 [3].

Tables 7-8 give the residuals between the above solutions after systematic differences due to the various coordinate systems and scales were removed. Tables 5-6 contain the transformation parameters and the corresponding variance-covariance matrices.

In conclusion, we can say as a result of this adjustment that the SAO and C-Band adjustments at least in the western hemisphere are compatible with one another and that the latter can successfully be used to establish the scale for the former. Our standard deviations are not much different than those obtained by the SAO using considerably more observational information and the meshing together of two totally independent adjustments created no undue stress.

Because of the weak Baker-Nunn connection across the Atlantic and the absence of a C-Band length which could be used in the eastern hemisphere, the results of our adjustments are weaker there than the SAO, but not excessively. The weakest point in the adjustment was Pretoria, South Africa which had optical observations only on a single line north to Addes Ababa and the C-Band length Merritt Island to Pretoria. For similar reasons, Tokyo is also poorly determined. The geometry of the network is not the best for geometric adjustments.

After the coordinate transformations between the SC 11 and SAO69 solutions (Tables 5-6), it seems that after the removal of the expected differences in the coordinate systems a small scale difference ($1 \pm .17$ ppm) is evident in the global network, which, considering that the SC 11 scale is based on radar measurements and the SAO69 on the adopted value of k^2M, is a very satisfactory agreement. On the other hand, in the European part of the network a large scale difference (-11 ± 2 ppm) is evident. None of the residuals in Tables 7-8 are statistically significant on the 3σ level, those on the 2σ level are boxed.

Further details on this and other solutions may be found in a separate report to be published soon.

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Figure 1

SAO 69 Simultaneous Global Observations

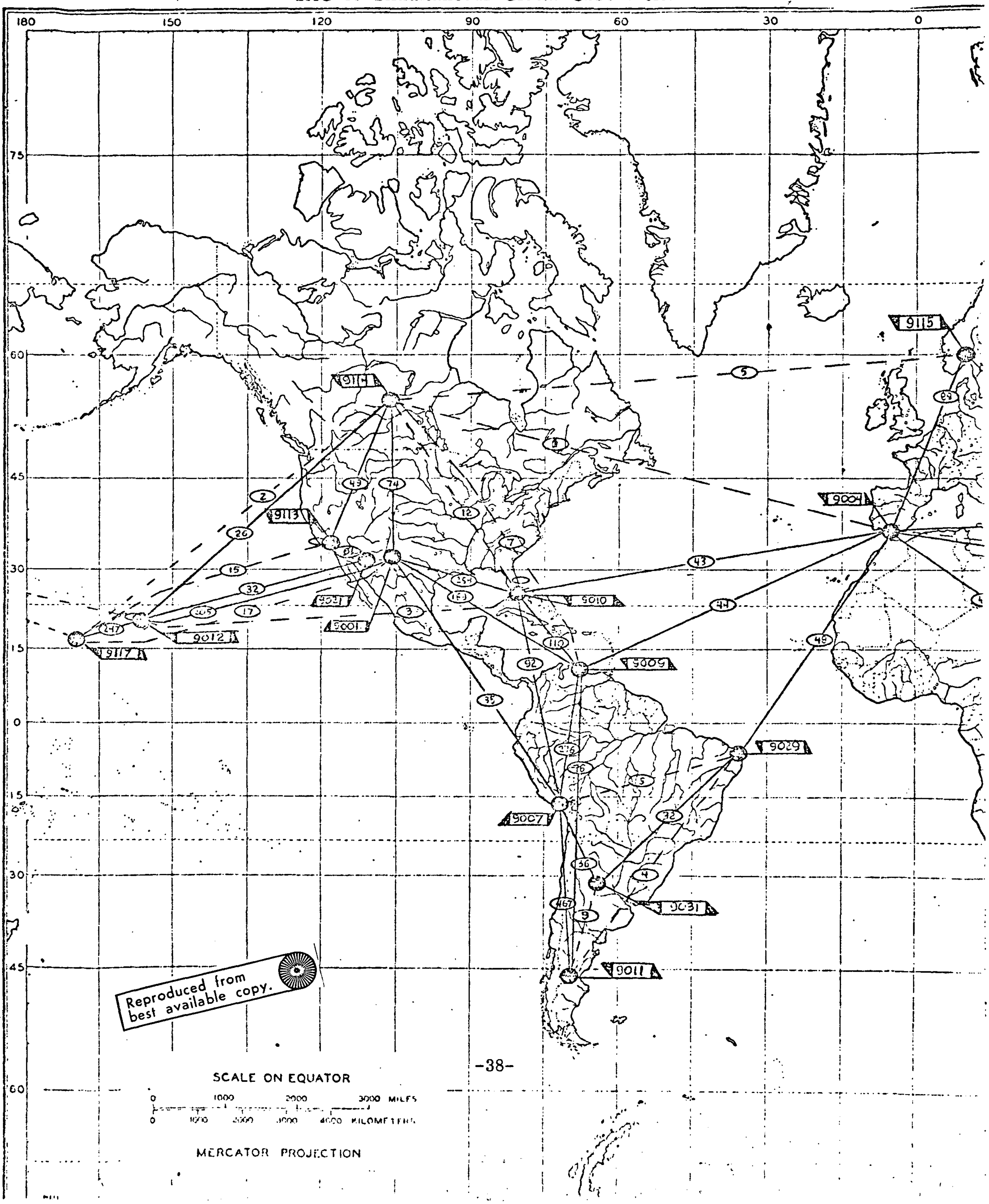

Figure 1 (continued)

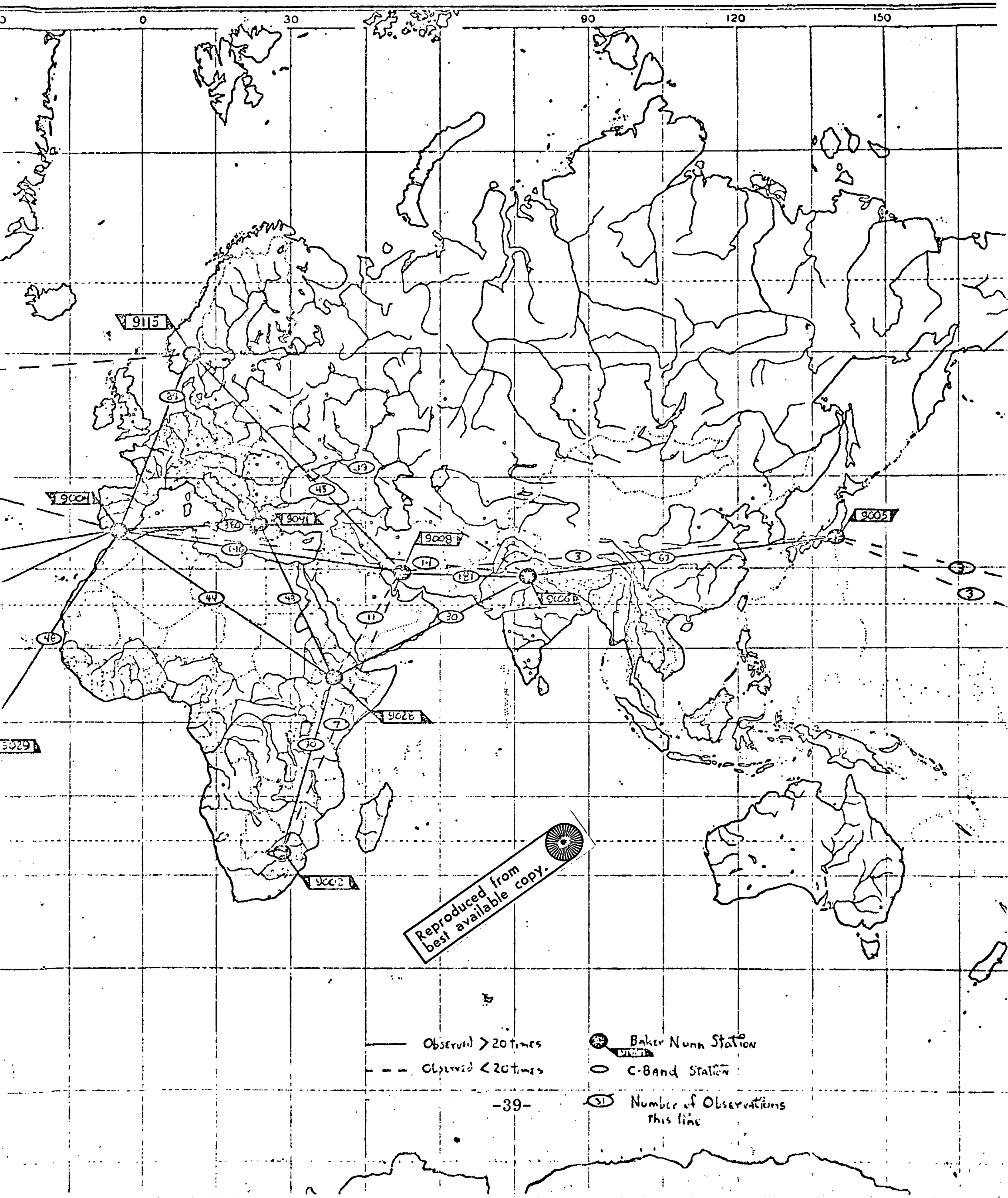

Figure 2

SAO 69 Simultaneous Observations in Europe

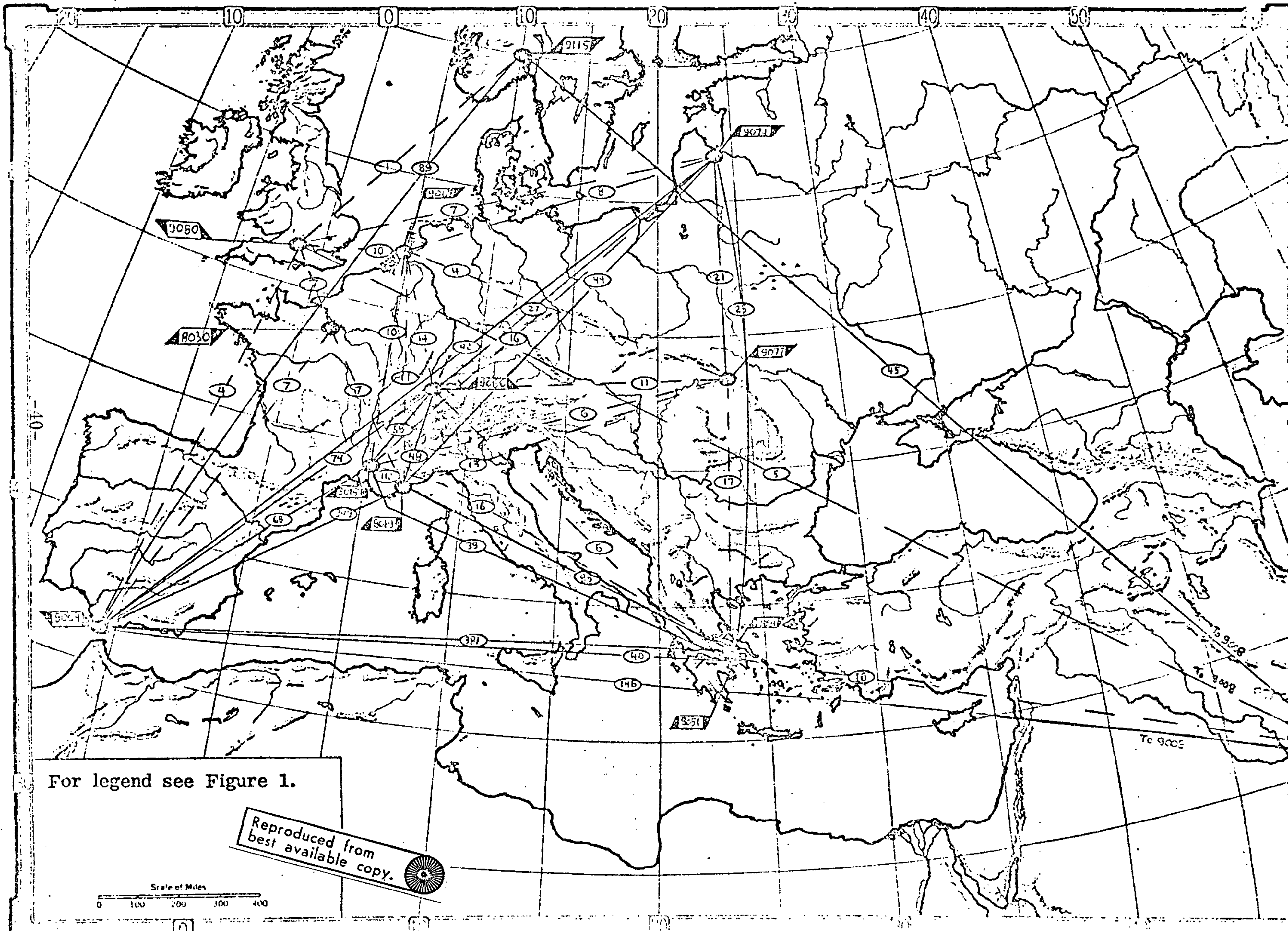

Table 1

SC 11 Coordinates

Station	X	Y	Z	Latitude	Longitude	Height
9001 ORGAN PASS, N.M.	-1535726.7818	-5167005.1570	3401065.4415	32 25 25.4459	253 26 40.5771	1625.8514
	8.1969	6.3418	8.9725	0.3419	0.3195	2.6892
9002 PRETORIA, S. AFRIC	5056149.1005	2716522.9256	-2775744.8458	- 25 57 35.0705	28 14 52.2764	1563.9394
	9.7327	23.9081	37.7128	1.3540	0.6637	3.2321
9004 SAN FERNANDO, SPAI	5105616.6396	-555231.7334	3769686.9984	36 27 46.5638	353 47 36.6712	92.5051
	5.8632	16.6043	6.6704	0.2627	0.6697	3.3081
9005 TOKYO, JAPAN	-3946689.0847	3366304.4797	3698858.5650	35 40 23.3166	139 32 15.6528	98.7805
	36.3804	33.5553	14.8014	0.5907	1.9169	3.6204
9006 NAINI TAL, INDIA	1018217.6706	5471104.6302	3109643.1581	29 21 35.1023	79 27 26.6317	1875.1719
	17.8904	8.4520	10.0332	0.3708	0.6858	4.4058
9007 AREQUIPA, PERU	1942808.3166	-5804084.2800	-1796890.8469	- 16 27 55.6887	288 30 25.2115	2477.2142
	8.4125	5.1977	8.6653	0.2944	0.3045	3.5758
9008 SHIRAZ, IRAN	3376918.4896	4403975.4597	3136270.8048	29 38 13.8284	52 31 9.9052	1582.2854
	9.3731	7.7237	9.4249	0.3437	0.3607	5.4689
9009 CURACAO, ANTILLES	2251857.6760	-5816916.4241	1327179.8606	12 5 25.3363	291 9 45.1023	-15.9949
	8.5344	4.9587	8.1786	0.2705	0.3081	2.9034
9010 JUPITER, FLA.	976315.9643	-5601392.3598	2880260.6007	27 1 14.6231	279 53 14.1319	-20.3870
	7.1646	5.2887	8.4464	0.3045	0.2760	2.2246
9011 VILLA DOLORES, ARG	2280625.6614	-4914569.5170	-3355380.3381	- 31 56 33.8999	294 53 37.8460	614.0428
	8.7289	8.1206	12.7995	0.4431	0.3842	4.2187
9012 MAUI, HAWAII	-5466061.6187	-2404281.1337	2242205.5918	20 42 26.6303	203 44 33.3302	3047.8636
	5.3050	6.0654	11.0734	0.3855	0.1979	3.7443
9021 MT. HOPKINS, ARIZ.	-1936843.9663	-5077658.9004	3331955.5296	31 41 4.3952	249 7 15.3618	2339.2166
	28.2621	19.7922	14.2635	0.5430	1.2329	7.5703
9028 ADDIS ABABA, ETHIO	4903748.7716	3965192.1028	963903.6656	8 44 52.4793	38 57 32.9440	1881.3827
	9.0439	10.4118	14.2038	0.4701	0.2763	10.5780
9029 NATAL, BRAZIL	5186502.5024	-3653844.0076	-654294.5160	- 5 55 39.1548	324 50 8.3453	39.9439
	11.2501	14.3439	22.4790	0.7351	0.5496	6.4718
9031 COMODORO RIVADAVIA	1693840.2611	-4112333.1051	-4556632.6095	- 45 53 11.9953	292 23 11.0703	183.9366
	11.9823	12.8399	21.0426	0.7168	0.5143	11.7670
9091 DIONYSOS, GREECE	4595195.1200	2039447.5045	3912679.9930	38 4 44.5210	23 55 57.6902	512.2695
	7.3741	15.5917	7.2629	0.3217	0.6054	5.8395
9113 ROSAMUND, CAL.	-2450009.8029	-4624434.0297	3635040.2114	34 57 50.5024	242 5 7.8204	755.4052
	12.1372	10.1253	12.2226	0.4741	0.5174	3.6649
9114 COLD LAKE, CANADA	-1264813.6820	-3466901.2539	5185480.3213	54 44 34.0956	249 57 24.2836	692.3276
	8.7187	9.7495	8.1114	0.3924	0.4942	3.3822
9115 HARESTUA, NORWAY	3121308.1290	592637.3104	5512721.6375	60 12 38.3382	10 45 2.4522	612.8235
	11.7376	14.2462	8.5093	0.4946	0.8395	3.4058
9117 JOHNSTON ISL., PAC	-6007420.9810	-1111827.2350	1825753.0404	16 44 39.0962	190 29 7.4691	19.2033
	9.5942	20.8791	13.4795	0.4645	0.7371	5.4007
9015 HAUTE PROVENCE, FR	4578361.9360	457975.7192	4403204.2349	43 55 57.0662	5 42 44.3591	716.3734
	7.6413	14.1732	9.4665	[illegible]	[illegible]	[illegible]

Table 1 (continued)

Station	X	Y	Z	Latitude	Longitude	Height
8019 NICE FRANCE	4579500.6319 7.1755	586608.1428 13.9056	4386427.8260 7.9417	43 43 32.5558 0.3458	7 17 58.2532 0.6086	430.2137 2.9016
8030 MEUDON FRANCE	4205665.5748 15.0483	163718.2754 18.4833	4776555.2889 13.5977	48 48 21.5631 0.6512	2 13 45.4303 0.8972	220.0017 3.6274
9051 ATHENS GREECE	4606897.0009 8.9424	2029688.3123 22.6786	3903572.3563 9.7282	37 58 36.8877 0.3932	23 46 37.6992 0.9365	222.8908 4.4120
9065 DELFT HOLLAND	3923467.8191 14.5774	299895.1507 16.0265	5003007.1170 13.9993	52 0 5.2871 0.5021	4 22 15.5101 0.8352	113.2129 13.0602
9066 ZIMMERWALD SWISS	4331347.9452 9.2708	567526.1262 14.1129	4633115.3721 9.7256	46 52 36.0462 0.4085	7 27 53.2914 0.6517	956.7412 5.4801
9080 MALVERN ENGLAND	3920225.5322 13.9462	-134771.9673 19.0970	5012770.4180 14.6714	52 8 35.1514 0.4691	358 1 51.6910 1.0129	210.2889 13.8990
9074 RIGA LATVIA	3183935.3143 15.8404	1421458.6318 14.5974	5322810.5529 13.4610	56 56 54.3586 0.5830	24 3 29.6417 0.9237	30.2361 8.5398
9077 UZGHÖRGD,USSR	3907461.3568 12.1981	1602411.9239 15.4160	4763928.3384 12.3368	48 38 1.2091 0.4529	22 17 53.0063 0.7551	241.3264 10.1886
4050 PRETORIA	5051649.1106 9.7342	2726617.4098 23.9078	-2774142.7911 37.7128	-25 56 36.5577 1.3544	28 21 28.3158 0.6625	1603.8506 3.2187
4082 MERRITT ISLAND	910605.6059 7.1646	-5539103.9400 5.2892	3017992.1100 8.4486	28 25 29.4952 0.3050	279 20 8.4550 0.2738	-25.3923 2.2353
4742 KAUAI H.I.	-5543968.7474 5.2630	-2054548.6706 6.2532	2387533.3608 11.2211	22 7 24.8854 0.3901	200 20 3.4309 0.2037	1146.3605 3.9179
4280 VANDENBERG AFB	-2671870.8347 12.2060	-4521213.3359 10.1610	3607494.1247 12.2294	34 39 56.8502 0.4756	239 25 6.5066 0.5235	88.0939 3.3978

Cartesian coordinates and heights in meters.

Geodetic coordinates refer to an ellipsoid of the following parameters:

$a = 6378155$ m $\qquad f = 1/298.255$

Note: Station numbers are those of SAO used in [2].

Table 2

Coordinate Differences SC 11 - SAO 69

	Station	ΔX	σ_X	ΔY	σ_Y	ΔZ	σ_Z	σ_{SAO}
Global Stations	9001	$\boxed{30.2}$	8.2	- 9.2	6.3	26.4	9.0	5
	9002	24.1	9.7	11.9	23.9	39.2	37.7	7
	9004	$\boxed{28.6}$	5.9	- 3.7	16.6	20.0	6.7	5
	9005	3.9	36.4	5.5	33.6	26.6	14.8	10
	9006	14.7	17.9	1.6	8.5	20.2	10.0	9
	9007	$\boxed{33.3}$	8.4	- 3.3	5.2	$\boxed{42.2}$	8.9	7
	9008	25.5	9.4	- 0.5	7.7	20.8	9.4	9
	9009	28.7	8.5	2.6	5.0	19.9	8.2	7
	9010	25.0	7.2	5.6	5.3	20.5	8.4	5
	9011	36.7	8.7	3.5	8.1	45.7	12.8	9
	9012	- 8.6	5.3	0.9	6.1	34.6	11.1	7
	9021	-62.0	28.3	45.1	19.8	39.5	14.3	15
	9028	- 1.2	9.0	- 8.9	10.4	31.7	14.2	12
	9029	41.5	11.3	12.0	14.3	30.5	22.5	12
	9031	37.3	12.0	- 5.1	12.8	16.4	21.0	15
	9091	$\boxed{38.1}$	7.4	22.5	15.6	$\boxed{29.9}$	7.3	5
	9113	1.2	12.1	-13.0	10.1	5.2	12.2	7
	9114	24.3	8.7	-17.3	9.7	13.3	8.1	12
	9115	28.1	11.7	- 5.7	14.2	20.6	8.5	17
	9117	-19.0	9.6	31.8	20.9	23.0	13.5	15
European Stations	8015	$\boxed{33.9}$	7.6	9.6	14.2	25.3	8.5	5
	8019	$\boxed{34.6}$	7.2	9.1	13.9	19.9	7.9	5
	9065	$\boxed{56.8}$	14.6	13.1	16.0	$\boxed{62.1}$	14.0	12
	9066	$\boxed{38.0}$	9.3	15.0	14.1	22.4	9.7	7
	9080	47.5	13.9	-34.0	19.1	$\boxed{62.4}$	14.7	9
	9074	34.3	15.8	10.7	14.6	38.6	13.5	10
	9077	40.4	12.2	14.9	15.4	38.3	12.3	10

All units are meters.

Table 3

Differences Between Distances SAO 69 - SC 11

	9001	9002	9004	9005	9006	9007	9008	9009	9010	9011	9012	9021	9028	9029	9031
9002	-3.9														
9004	-1.5	10.1													
9005	-21.2	-8.6	-26.3												
9006	-7.0	13.3	-12.2	-9.0											
9007	11.9	-11.4	17.0	-13.7	2.9										
9008	-5.7	20.7	-3.9	-20.9	-10.8	7.8									
9009	-0.0	1.2	5.1	-17.3	-0.6	22.7	3.2								
9010	6.4	5.2	5.0	-10.4	4.1	19.0	5.9	-3.2							
9011	13.2	-3.7	26.9	-6.8	12.7	-0.9	18.5	25.1	22.3						
9012	-34.6	-29.7	-33.5	-5.5	-14.1	-32.3	-25.0	-34.8	-26.6	-28.6					
9021	-98.2	-28.5	-48.8	54.3	20.8	-60.5	-3.3	-95.8	-94.3	-46.5	66.0				
9028	18.6	12.7	9.4	7.3	15.1	17.6	22.6	19.3	25.0	26.4	-1.5	4.7			
9029	-11.4	2.5	17.4	-22.8	2.0	-11.4	12.0	-8.9	-8.1	3.9	-47.9	-87.6	18.0		
9031	-12.4	-14.4	-0.6	-28.1	-9.8	-20.6	-4.6	-0.4	-3.2	-17.9	-45.1	-56.6	6.8	-14.8	
9091	-29.4	11.2	-24.4	-31.3	-4.0	-14.4	11.6	-23.8	-21.8	-2.0	-50.2	-50.4	22.0	-8.1	-26.4
9113	-17.9	-11.2	-28.9	-17.9	-17.5	10.0	-22.9	-14.3	-14.2	14.2	-4.9	92.4	4.6	-25.4	-6.7
9114	15.6	2.2	-3.0	-25.4	-14.0	25.6	-13.0	9.9	16.6	28.1	-18.1	35.8	16.4	-0.6	-0.0
9115	0.3	14.1	0.0	-24.5	-11.2	18.3	-4.1	6.5	9.0	26.7	-25.9	-16.0	19.1	13.9	-2.4
9117	-62.7	-26.0	-44.6	12.1	-3.5	-51.5	-21.5	-55.9	-50.4	-43.0	-34.4	34.5	1.7	-60.3	-56.1
9074	-20.8	2.6	-14.7	-29.9	-9.1	-7.5	-1.0	-16.7	-12.9	1.7	-42.3	-31.4	15.2	-7.0	-26.5
9077	-26.7	3.7	-16.7	-34.8	-10.4	-12.2	2.1	-20.9	-18.2	-2.0	-50.0	-43.9	15.8	-7.7	-29.2
4050	-13.6	-4.5	16.1	-14.3	17.6	-25.3	30.1	-7.4	-2.7	-19.2	-42.6	-38.5	20.5	-3.3	-33.1
4082	-2.4	9.0	10.2	-14.8	3.9	21.4	7.7	2.8	4.3	24.7	-34.3	-102.5	28.5	-1.0	-1.8
4280	-29.6	-7.8	-38.1	-12.1	-17.1	12.9	-25.9	-19.5	-23.7	19.6	13.0	54.8	3.6	-27.9	1.8
4742	-4.4	0.6	3.5	5.2	10.2	-3.0	5.0	-0.9	7.3	-0.6	6.5	96.2	30.3	-12.1	-21.5

	9091	9113	9114	9115	9117	9074	9077	4050	4082	4280
9113	-17.9									
9114	15.6	2.2								
9115	0.3	14.1	0.0							
9117	-62.7	-26.0	-44.6	12.1						
9074	-20.8	2.6	-14.7	-29.9	-9.1					
9077	-26.7	3.7	-16.7	-34.8	-10.4	-12.2				
4050	-13.6	-4.5	14.1	-14.3	17.6	-25.3	30.1			
4082	-2.4	9.0	10.2	-14.8	3.9	21.4	7.7	2.8		
4280	-29.6	-7.8	-38.1	-12.1	-17.1	12.9	-25.8	-19.5	-23.7	
4742	-4.4	0.6	3.5	5.2	10.2	-3.0	5.0	-0.9	7.3	-0.6

All distances in meters.

Table 4

Differences Between Distances GSFC 71 - SC 11

	9001	9002	9004	9005	9006	9007	9008	9009	9010	9011	9012	9021	9028	9029	9031
9002	9.5														
9004	-13.3	36.6													
9005	-21.3	4.1	-24.0												
9006	9.9	-4.6	7.7	10.0											
9007	25.1	-9.9	23.0	-3.4	9.8										
9008	-4.9	45.1	17.0	-29.6	-24.5	25.8									
9009	3.6	10.8	-9.7	-15.6	5.5	38.8	7.8								
9010	-0.0	9.5	-4.9	-18.2	12.2	21.3	6.8	-6.4							
9011	18.1	9.8	32.6	0.6	12.4	-9.5	36.0	31.0	14.3						
9012	-39.3	-13.2	-37.1	3.2	7.3	-26.6	-21.9	-34.1	-41.8	-26.3					
9021	-94.1	-13.7	-55.8	54.8	40.1	-48.3	0.3	-90.3	-98.6	-42.7	58.0				
9028	39.5	3.0	41.4	34.4	19.7	30.2	65.0	28.6	37.5	35.7	26.6	28.5			
9029	18.0	12.3	63.3	11.3	31.8	-19.7	59.5	8.6	17.9	-17.9	-24.4	-57.0	45.3		
9031	-0.8	8.6	21.1	-12.4	-1.5	-22.2	26.5	14.6	-4.2	-10.1	-39.2	-47.0	27.5	-25.2	
9091	3.5	29.0	13.1	-17.1	-0.2	28.2	4.9	6.3	12.4	35.7	-17.3	-13.3	35.9	60.7	23.7
9113	-17.1	-13.1	-47.5	-29.1	-12.0	13.0	-35.6	-16.0	-22.9	7.3	-18.6	92.1	13.4	-5.3	-8.9
9114	-0.8	8.1	-11.8	-37.5	6.1	26.8	-14.4	6.5	13.7	19.6	-44.6	21.4	39.2	34.8	-2.2
9115	-12.1	51.9	12.5	-25.9	27.1	35.4	17.5	1.8	3.8	43.0	-31.0	-23.7	71.0	73.7	28.9
9117	-59.9	-19.9	-50.9	5.3	1.4	-43.0	-31.4	-51.0	-60.3	-39.3	-19.9	35.9	19.4	-38.5	-49.3
9074	-13.7	31.6	26.5	-58.4	-11.2	28.8	-12.7	5.9	8.0	33.3	-48.1	-21.5	50.3	72.7	15.4
9077	0.6	24.2	23.1	-41.2	-15.7	32.6	-19.2	11.5	16.6	36.8	-31.0	-13.3	34.0	70.6	20.4
4050	8.2	-0.2	34.6	2.9	-5.9	-1.6	43.4	9.6	8.1	9.5	-14.1	-14.9	1.3	11.3	8.6
4082	-1.9	9.4	-7.2	-20.9	10.1	24.1	4.1	-4.0	3.3	16.7	-43.1	-99.4	35.4	20.0	-2.1
4280	-19.1	-14.6	-49.4	-29.8	-13.1	10.8	-37.2	-17.0	-24.0	5.3	-18.3	90.9	12.4	-7.0	-10.8
4742	-39.9	-13.8	-38.4	1.9	6.1	-26.3	-24.3	-34.4	-41.8	-25.8	0.0	58.5	25.6	-23.6	-38.9

	9091	9113	9114	9115	9117	9074	9077	4050	4082	4280
9113	-17.1									
9114	-0.8	9.1								
9115	-12.1	51.9	12.5							
9117	-55.9	-19.9	-50.9	5.3						
9074	-13.7	31.6	26.5	-58.4	-11.2					
9077	0.6	24.2	23.1	-41.2	-15.7	32.6				
4050	8.2	-0.2	34.6	2.9	-5.9	-1.6	43.4			
4082	-1.9	9.4	-7.2	-20.9	10.1	24.1	4.1	-4.0		
4280	-19.1	-14.6	-49.4	-29.8	-13.1	10.8	-37.2	-17.0	-24.0	
4742	-39.9	-13.8	-33.4	1.9	6.1	-26.3	-24.3	-34.4	-41.8	-25.8

All distances in meters.

Table 5

Global Transformations SC 11 - SAO 69

ΔX(m)	ΔY(m)	ΔZ(m)
22.07	-0.04	24.79
1.0071	-0.0000	-0.
-0.0000	1.0198	0.
-0.	0.	1.0857

ΔX(m)	ΔY(m)	ΔZ(m)	θ_z (")	θ_y (")	θ_x (")	ϵ ($\times 10^{-6}$)
20.99	1.65	21.40	-0.12	0.14	0.03	1.04
1.5953	-0.0257	-0.1436	0.0237	0.0312	0.0043	-0.0419
-0.0257	1.4268	0.0479	0.0094	-0.0033	-0.0208	0.0755
-0.1436	0.0479	1.5961	-0.0011	-0.0222	-0.0244	-0.0679
0.0237	0.0094	-0.0011	0.0020	0.0004	-0.0001	-0.0002
0.0312	-0.0033	-0.0222	0.0004	0.0028	0.0005	-0.0002
0.0043	-0.0208	-0.0244	-0.0001	0.0005	0.0023	-0.0000
-0.0419	0.0755	-0.0679	-0.0002	-0.0002	-0.0000	0.0325

All rotations are positive for counterclockwise rotation as viewed looking toward the origin from the positive end of the rotation axes as follows:

x	parallel to 0 meridian
y	parallel to 90°E meridian
z	parallel to rotation axis of the earth (ICO)

Table 6

European Transformations SC 11 - SAO 69

ΔX(m)	ΔY(m)	ΔZ(m)
39.24	6.54	31.52
2.5796	-0.0000	-0.
-0.0000	3.2929	0.
-0.	0.	2.9329

ΔX(m)	ΔY(m)	ΔZ(m)	θ_z('')	θ_y('')	θ_x('')	ϵ ($\times 10^{-6}$)
-101.73	-88.92	77.94	-1.86	-4.44	2.21	10.95
332.6312	137.3697	-125.0857	1.1341	10.5583	-4.3210	-22.2762
137.3698	495.3402	-148.6948	9.2333	6.3285	-13.2196	-5.3436
-125.0862	-148.6950	328.1671	-2.2454	-9.9465	5.1874	-22.5386
1.1341	9.2333	-2.2454	0.3043	0.0899	-0.1372	-0.0107
10.5583	6.3285	-9.9465	0.0899	0.4675	-0.1956	-0.0471
-4.3210	-13.2196	5.1874	-0.1372	-0.1956	0.4533	0.0714
-22.2762	-5.3435	-22.5387	-0.0107	-0.0471	0.0714	5.0751

Table 7

Global Residuals SC 11 - SAO 69

Station	Transf. 3 Param.			Transf. 7 Param.		
	ΔX	ΔY	ΔZ	ΔX	ΔY	ΔZ
9001	- 8.1	9.1	- 1.6	-10.2	5.0	- 1.8
9004	- 6.6	3.7	4.8	- 4.6	8.2	8.9
9006	7.4	- 1.7	4.6	2.2	6.7	4.4
9007	-11.2	3.2	-17.4	- 5.8	- 0.3	-20.5
9008	- 3.4	0.5	4.0	- 5.6	9.1	5.5
9009	- 6.6	- 2.6	4.9	- 2.9	- 5.5	5.2
9010	- 2.9	- 5.7	4.2	- 1.7	- 8.9	5.2
9011	-14.6	- 3.5	-20.9	- 8.2	- 6.1	-25.6
9012	30.7	- 0.9	- 9.8	23.7	- 4.5	-14.2
9028	23.3	8.9	- 6.9	24.5	17.6	- 6.5
9029	-19.4	-12.0	- 5.7	-12.6	-11.3	- 5.8
9031	-15.2	5.1	8.4	- 9.1	2.8	2.0
9091	-16.0	-22.5	- 5.1	-16.1	-15.6	- 1.6
9113	20.9	13.0	19.6	17.4	9.0	19.0
9114	- 2.2	17.2	11.5	- 6.2	15.3	13.1
9115	- 6.1	5.6	4.2	- 7.9	10.5	8.5
9117	41.1	-31.8	1.8	33.1	-34.4	- 3.6

All distances in meters.

Table 8

European Residuals SC 11 - SAO 69

Station	Transf. 3 Param.			Transf. 7 Param.		
	ΔX	ΔY	ΔZ	ΔX	ΔY	ΔZ
8015	5.3	- 3.1	6.3	5.1	- 5.1	- 2.4
8019	4.6	- 2.7	11.7	2.9	- 3.5	1.4
9065	-17.6	- 6.6	-30.6	-10.7	- 9.8	-17.0
9066	+ 1.3	- 8.5	9.1	2.3	- 9.1	7.1
9074	5.0	- 4.2	- 7.1	0.5	1.7	14.0
9077	- 1.1	- 8.4	- 6.8	-11.3	- 0.0	- 9.4
9080	- 8.3	40.5	-30.9	2.7	32.6	-12.4

All distances in meters.

2.4 Geodetic Satellite Observations in North America Solution NA-8

The results of the adjustments of the GEOS-I tracking network through the NA-6 adjustment were reported in [1]. After the completion of these adjustments, it was apparent that the weakest of the adjusted station coordinates were the heights. The approximate height coordinates used in the adjustments were taken from the station descriptions on the geodetic data sheets of [2]. The only height constraint imposed was at Columbia, Missouri; all others were allowed to adjust freely. At the time, it was not possible to constrain any station height with any degree of accuracy.

After the completion of all previous adjustments, a new geoid became available from SAO [3]. This geoid gave the heights above the SAO ellipsoid. This ellipsoid is earth-centered, and based on comparison of station coordinates in the continental United States, the following shifts were determined for the North American Datum:

$$\Delta x = -38 \text{ m}$$
$$\Delta y = 164 \text{ m}$$
$$\Delta z = 175 \text{ m}$$

The sign convention of these shifts is SAO-NAD.

With the geoid map, it was possible to determine the geoid undulations at each of the observing stations in the optical network. Since the orthometric heights were well determined at the stations, it was simply a matter of adding the geoid height and the orthometric height to arrive at heights with respect to the SAO ellipsoid. By performing a datum transformation the heights were computed with respect to the Clark 1866 ellipsoid, i.e., with respect to the NAD.

The NA-6 solution was readjusted with the new computed heights as constraints placed on all 30 optical stations, using a weight corresponding to a standard deviation of 5 meters. This was referred to as the NA-8 solution; the results are listed in Table 1.

The NA-8 solution shows the adjusted coordinates to be realistic, the standard deviations of the adjusted coordinates being smaller than those of any other OSU adjustment. The NA-8 coordinates cannot be compared directly with the NAD coordinates because of the height change at the origin. In order to make a comparison with the NAD coordinates, the following coordinate differences must be added to the NA-8 coordinates:

$$\Delta x = -1.6 \text{ meters}$$

$$\Delta y = +29.4 \text{ meters}$$

$$\Delta z = -20.5 \text{ meters}$$

These are the shifts of Columbia, Missouri from its NAD coordinates. The NAD geodetic coordinates of the stations are listed in Table 2.

Table 3 shows the datum transformation parameters NA8-NAD. The interpretation of the parameters are identical to those in [1]. Comparisons were also made with other solutions, namely, those of the Smithsonian Astrophysical Observatory [3], the Goddard Space Flight Center [4], and our previous NA-6 [1].

The comparisons show that after removing the systematic differences due to the different coordinate systems and scale (Table 4), the residuals are smaller than those expected from the standard deviations on the 3σ level (Table 5). Those residuals which are greater than 2σ (but still smaller than 3σ) are framed in Table 5. 2σ in this case corresponds approximately to a confidence of 95%.

More details and other computations will be published in a separate report.

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[1] Mueller, Ivan I., James P. Reilly and Charles R. Schwarz (1969, revised January, 1970). "The North American Datum in View of GEOS-I Observations," The Ohio State University, Department of Geodetic Science Report No. 125.

[2] NASA/Goddard Space Flight Center. (1968). Geodetic Satellites Observation Station Directory. Compiled by Geonautics, Inc. July 1968.

[3] Gaposchkin, E. M. and K. Lambeck. (1970). "1969 Smithsonian Standard Earth (II)", SAO Special Report No. 315. Smithsonian Astrophysical Observatory, Cambridge, Massachusetts.

[4] Marsh, J. G., B. C. Douglas and S. M. Klosko. (1971). "A Unified Set of Tracking Station Coordinates Derived from Geodetic Satellite Tracking Data", Goddard Space Flight Center, Greenbelt, Maryland, Report No. X-553-71-370.

Table 1

Coordinates of the North American GEOS-I Tracking Stations from the NA-8 Geometric Adjustment

Station	Name		NA-8	σ
7075	Sudbury, Ontario	X	692, 651. 9	5. 0
	MOTS 40	Y	-4, 347, 244. 3	5. 2
		Z	4, 600, 327. 8	4. 6
1032	St. Johns,	X	2, 602, 711. 4	69. 0
	Newfoundland	Y	-3, 419, 407. 3	82. 6
	MOTS 40	Z	4, 697, 483. 9	23. 0
3334	Greenville,	X	-84, 946. 0	5. 9
	Mississippi	Y	-5, 328, 137. 9	5. 7
	PC-1000	Z	3, 493, 288. 8	6. 7
3902	Cheyenne,	X	-1, 234, 655. 7	10. 3
	Wyoming	Y	-4, 651, 395. 3	7. 8
	PC-1000	Z	4, 174, 601. 5	6. 9
1033	College, Alaska	X	-2, 299, 235. 3	14. 8
	MOTS 40	Y	-1, 445, 862. 1	40. 1
		Z	5, 751, 645. 5	7. 6
3400	Colorado Springs,	X	-1, 275, 164. 6	10. 5
	Colorado	Y	-4, 798, 189. 8	6. 3
	PC-1000	Z	3, 994, 051. 2	6. 4
3903	Herndon,	X	1, 089, 024. 2	7. 0
	Virginia	Y	-4, 843, 181. 5	4. 7
	PC-1000	Z	3, 991, 553. 2	5. 2
7039	Bermuda	X	2, 308, 259. 6	7. 3
	Island	Y	-4, 873, 768. 5	4. 5
	MOTS 40	Z	3, 394, 403. 8	5. 1

Station	Name		NA-8	σ
3405	Grand Turk	X	1, 919, 533. 3	6. 6
	PC-1000	Y	-5, 621, 259. 5	3. 9
		Z	2, 315, 614. 3	7. 0
3407	Trinidad	X	2, 979, 938. 0	9. 8
	PC-1000	Y	-5, 513, 709. 1	4. 8
		Z	1, 180, 999. 5	11. 9
3648	Hunter AFB,	X	832, 602. 0	4. 6
	Georgia	Y	-5, 349, 700. 7	2. 9
	PC-1000	Z	3, 360, 426. 0	3. 8
3404	Swan Island	X	642, 530. 2	5. 3
	PC-1000	Y	-6, 054, 110. 8	4. 0
		Z	1, 895, 536. 4	7. 2
3657	Aberdeen,	X	1, 186, 826. 5	4. 9
	Maryland	Y	-4, 785, 353. 7	4. 3
	PC-1000	Z	4, 032, 731. 5	4. 0
3406	Curacao	X	2, 251, 848. 0	6. 8
	PC-1000	Y	-5, 817, 084. 0	3. 7
		Z	1, 327, 048. 4	9. 8
7076	Jamaica,	X	1, 384, 198. 3	5. 9
	B.W.I..	Y	-5, 905, 837. 1	4. 3
	MOTS 40	Z	1, 966, 390. 9	7. 7
1021	Blossom Point,	X	1, 118, 060. 3	4. 8
	Maryland	Y	-4, 876, 485. 7	3. 8
	MOTS 40	Z	3, 942, 816. 1	4. 0

All coordinates and standard deviations in meters.

Table 1 continued

Station	Name		NA-8	σ
3402	Semmes,	X	167,294.5	4.4
	Alabama	Y	-5,482,137.8	3.3
	PC-1000	Z	3,244,883.8	4.5
3401	L.G.Hanscom Field,	X	1,513,177.0	5.4
	Mass.	Y	-4,463,733.1	5.0
	PC-1000	Z	4,282,902.6	4.0
3106	Antiqua Island	X	2,881,885.5	8.4
	PC-1000	Y	-5,372,339.3	3.8
		Z	1,868,386.0	7.8
3861	Homestead AFB,	X	961,808.8	4.4
	Florida	Y	-5,679,324.0	2.9
	PC-1000	Z	2,729,731.8	4.1
7040	San Juan,	X	2,465,099.4	7.3
	P.R.	Y	-5,535,102.1	3.6
	MOTS40	Z	1,985,359.7	6.9
7043	GSFC,Greenbelt,	X	1,130,748.4	4.7
	Maryland	Y	-4,831,490.0	3.7
	PTH-100	Z	3,993,979.6	3.9
7045	Denver,	X	-1,240,436.8	5.4
	Colorado	Y	-4,760,404.0	3.9
	MOTS40	Z	4,048,826.0	4.1
1042	Rosman, N.C.	X	647,534.9	4.0
	MOTS40	Y	-5,178,101.3	3.0
		Z	3,656,553.1	3.9
7072	Jupiter,Florida	X	976,303.4	4.4
	MOTS40	Y	-5,601,569.2	3.1
		Z	2,880,088.5	4.5

General Information:

No. of ground stations 34

No. of spatial chord equations 1

Station	Name		NA-8	σ
7036	Edinburg,	X	-828,460.7	4.4
	Texas	Y	-5,657,636.6	3.3
	MOTS40	Z	2,816,659.3	4.9
1034	E. Grand Fork	X	-521,674.7	3.7
	Minn.	Y	-4,242,224.0	4.4
	MOTS40	Z	4,718,565.8	4.0
1030	Mojave,	X	-2,357,218.4	7.9
	California	Y	-4,646,496.9	4.0
	MOTS40	Z	3,668,147.4	4.1
7037	Columbia.	X	-191,259.0	2.3
	Missouri	Y	-4,967,457.8	2.7
	MOTS40	Z	3,983,105.0	2.9
1022	Ft. Myers,	X	807.890.9	3.9
	Florida	Y	-5,652,159.6	2.7
	MOTS40	Z	2,833,347.1	4.3
5861	Homestead,	X	963,509.5	4.4
	Florida	Y	-5,679,888.6	2.9
	SECOR	Z	2,727,970.8	4.1
5001	Herndon,	X	1,088,884.1	7.0
	Virginia	Y	-4,843,066.3	4.7
	SECOR	Z	3,991,662.7	5.2
5333	Stoneville,	X	-84,964.3	5.9
	Mississippi	Y	-5,328,135.5	5.7
	SECOR	Z	3,493,297.7	6.7
5649	Hunter AFB,	X	832,519.4	4.6
	Georgia	Y	-5,349,740.3	2.9
	SECOR	Z	3,360,381.1	3.8

No. of degrees of freedom 5254

Quadratic sum of the residuals ($V'PV$) 5082

Standard deviation of unit weight 1.0

Table 2

NAD Coordinates of the North American GEOS-I Tracking Stations

Station	Name		NAD	σ	Station	Name		NAD	σ
7075	Sudbury, Ontario	φ	46° 27′ 21″.42	0″.18	3405	Grand Turk	φ	21 25 46.51	0.23
	MOTS40	λ	279 3 10.57	0″.25		PC-1000	λ	288 51 14.18	0.23
		h	279.9m	3.7m			h	-13.4	3.7
1032	St. Johns,	φ	47 44 29.32	1.10	3407	Trinidad	φ	10 44 31.98	0.39
	Newfoundland	λ	307 16 37.53	5.02		PC-1000	λ	298 23 22.33	0.33
	MOTS40	h	99.0	4.9			h	265.3	4.0
3334	Greenville,	φ	33 25 31.36	0.25	3648	Hunter AFB,	φ	32 0 5.93	0.14
	Mississippi	λ	269 5 11.68	0.23		Georgia	λ	278 50 46.63	0.17
	PC-1000	h	41.0	4.4		PC-1000	h	-2.4	2.1
3902	Cheyenne,	φ	41 7 58.28	0.30	3404	Swan Island	φ	17 24 16.78	0.24
	Wyoming	λ	255 8 3.18	0.43		PC-1000	λ	276 3 29.48	0.18
	PC-1000	h	1875.0	4.9			h	54.3	3.5
1033	College, Alaska	φ	64 52 19.81	0.50	3657	Aberdeen,	φ	39 28 19.45	0.15
	MOTS40	λ	212 9 46.65	3.05		Maryland	λ	283 55 44.58	0.21
		h	159.2	4.9		PC-1000	h	-3.6	3.4
3400	Colorado Springs,	φ	39 0 22.41	0.26	3406	Curacao	φ	12 5 22.81	0.32
	Colorado	λ	255 7 1.19	0.43		PC-1000	λ	291 9 43.04	0.22
	PC-1000	h	2181.0	4.7			h	33.2	3.5
3903	Herndon,	φ	38 59 32.41	0.19	7076	Jamaica,	φ	18 4 32.23	0.26
	Virginia	λ	282 40 21.48	0.29		B.W.I.	λ	283 11 26.9	0.20
	PC-1000	h	116.2	4.2		MOTS40	h	471.0	3.9
7039	Bermuda	φ	32 21 48.89	0.18	1021	Blossom Point,	φ	38 25 50.00	0.14
	Island	λ	295 20 33.89	0.29		Maryland	λ	282 54 48.21	0.20
	MOTS40	h	17.8	3.7		MOTS40	h	-4.5	3.1

Note: The above coordinates were arrived at by applying the shifts $\Delta X = -1.6$ m, $\Delta Y = 29.4$ m and $\Delta Z = -20.5$ m to the NA-8 coordinates and then converting these values to ellipsoidal coordinates on the ellipsoid $a = 6378206.4$, $b = 6356583.8$.

Table 2 continued

Station	Name		NAD	σ
3402	Semmes,	φ	30 46 49.57	0.15
	Alabama	λ	271 44 52.40	0.17
	PC-1000	h	68.5	3.0
3401	L.G.Hanscom Field,	φ	42 27 18.29	0.16
	Mass.	λ	288 43 35.07	0.26
	PC-1000	h	71.8	3.5
3106	Antiqua Island	φ	17 8 52.39	0.26
	PC-1000	λ	298 12 38.00	0.28
		h	2.5	3.3
3861	Homestead AFB,	φ	25 30 25.02	0.14
	Florida	λ	279 36 43.24	0.16
	PC-1000	h	3.6	2.6
7040	San Juan,	φ	18 15 26.06	0.23
	P.R.	λ	294 0 22.47	0.24
	MOTS40	h	51.2	3.1
7043	GSFC,Greenbelt,	φ	39 1 15.69	0.14
	Maryland	λ	283 10 20.30	0.20
	PTH-100	h	39.1	3.4
7045	Denver,	φ	39 38 48.21	0.14
	Colorado	λ	255 23 41.53	0.23
	MOTS40	h	1791.0	3.3
1042	Rosman, N.C.	φ	35 12 7.10	0.13
	MOTS40	λ	277 7 40.76	0.16
		h	904.9	2.7
7072	Jupiter,Florida	φ	27 1 13.18	0.16
	MOTS40	λ	279 53 12.64	0.16
		h	16.9	2.7

Station	Name		NAD	σ
7036	Edinburg,	φ	26 22 45.37	0.17
	Texas	λ	261 40 9.04	0.16
	MOTS40	h	68.2	3.0
1034	E. Grand Fork	φ	48 1 21.53	0.16
	Minn.	λ	262 59 21.60	0.19
	MOTS40	h	253.7	2.9
1030	Mojave,	φ	35 19 47.97	0.14
	California	λ	243 6 2.32	0.32
	MOTS40	h	898.8	3.2
7037	Columbia,	φ	38 53 36.07	0.09
	Missouri	λ	267 47 42.06	0.09
	MOTS40	h	272.9	2.7
1022	Ft. Myers,	φ	26 32 51.95	0.15
	Florida	λ	278 8 4.12	0.14
	MOTS40	h	15.0	2.1
5861	Homestead,	φ	25 29 21.60	0.14
	Florida	λ	279 37 39.90	0.16
	SECOR	h	4.6	2.6
5001	Herndon,	φ	38 59 38.09	0.19
	Virginia	λ	282 40 16.85	0.29
	SECOR	h	73.9	4.2
5333	Stoneville,	φ	33 25 31.64	0.25
	Mississippi	λ	269 5 10.97	0.23
	SECOR	h	44.2	4.4
5649	Hunter AFB,	φ	32 0 4.24	0.14
	Georgia	λ	278 50 43.29	0.17
	SECOR	h	-3.8	2.1

Table 3

Datum Transformation Parameters: NA-8 - NAD

		Eastern Half (14 stations)*	Western Half (5 stations)**	Combination (19 stations)
Veis	θ_z (")	-1.2 ± 0.4	0.8 ± 0.8	-1.0 ± 0.3
	θ_y (")	-0.2 ± 0.4	1.0 ± 0.9	-0.1 ± 0.4
	θ_x (")	1.6 ± 0.5	-0.6 ± 1.0	0.6 ± 0.2
Molo-densky	θ_z (")	-2.0 ± 0.5	0.9 ± 1.0	-1.1 ± 0.3
	θ_y (")	0.0 ± 0.4	-0.4 ± 1.0	0.4 ± 0.3
	θ_x (")	-0.2 ± 0.4	1.0 ± 0.9	-0.1 ± 0.4
	ϵ ($\times 10^{-6}$)	1.3 ± 1.9	-0.4 ± 4.2	0.1 ± 1.4
Shifts	ΔX(m)	1.0 ± 2.9	8.2 ± 4.6	3.1 ± 2.0
	ΔY(m)	-31.1 ± 3.2	-28.8 ± 4.5	-25.3 ± 1.7
	ΔZ(m)	20.8 ± 3.1	12.6 ± 4.2	17.0 ± 1.7

*Eastern Stations: 1021, 1022, 1034, 1042, 3334, 3401, 3402, 3648, 3657, 3861, 3037, 7043, 7072, 7075.

**Western Stations: 1030, 3400, 3902, 7036, 7045.

All rotations are about Meades Ranch and are positive for counterclockwise rotation as viewed looking toward the origin from the positive end of the rotation axes as follows:

Veis:
- x horizon plane south
- y horizon plane east
- z zenith

Molodensky:
- x parallel to 0 meridian
- y parallel to 90°E meridian
- z parallel to rotation axis of the earth (ICO)

Table 4

Transformations Between Various Solutions

Transformation Parameters	NA 8 - NA 6		NA 8 - GSFC		NA 8 - SAO 69	
	Transf. 3 Param.	Transf. 7 Param.	Transf. 3 Param.	Transf. 7 Param.	Transf. 3 Param.	Transf. 7 Param.
ΔX (m)	-2.2 ± 1.8	4.7 ± 2.4	-26.4 ± 1.3	-25.0 ± 1.8	-27.3 ± 2.9	-31.6 ± 3.8
ΔY (m)	-27.2 ± 1.9	-31.4 ± 2.4	160.8 ± 2.0	155.6 ± 2.6	150.3 ± 3.0	144.2 ± 4.0
ΔZ (m)	21.3 ± 1.9	22.3 ± 2.4	172.2 ± 1.8	177.3 ± 2.3	176.1 ± 3.0	183.8 ± 3.9
θ_z ('')		0.47 ± 0.29		0.28 ± 0.25		0.47 ± 0.45
θ_y ('')		-1.00 ± 0.54		-0.52 ± 0.49		-1.64 ± 0.75
θ_x ('')		0.63 ± 0.33		1.22 ± 0.30		1.18 ± 0.56
ϵ ($\times 10^{-6}$)		-6.00 ± 1.37		0.24 ± 0.94		5.05 ± 2.01

Rotations About Meades Ranch (θ_x Rotation in the PV Plane to West; θ_y Rotations in the Meridian Plane to South; θ_z Rotation in Azimuth to West).

In these transformations, the shifts mentioned on page 51 have already been taken into account, i.e., the correct heading should be for example (NA 8 $\rightarrow$ NAD) - GSFC, etc.

Table 5

Station-coordinate Residuals After Transformations

		NA 8 - NA 6		NA 8 - GSFC		NA 8 - SAO69					
		Transf. 3 Param.	Transf. 7 Param.	Transf. 3 Param.	Transf. 7 Param.	Transf. 3 Param.	Transf. 7 Param.	σ_{NA6}	σ_{GSFC} (mean)	σ_{SAO}	σ_{NA8}
1021	ΔX	5.7	1.2	- 1.7	- 0.2	2.4	7.1	5.7	2.8	7.0	4.8
	ΔY	- 2.8	- 5.0	$\boxed{16.5}$	$\boxed{18.4}$	10.0	10.1	5.3	5.3	7.0	3.8
	ΔZ	2.0	- 1.9	1.7	- 1.2	-12.4	-12.5	4.9	4.2	7.0	4.0
1022	ΔX	4.2	- 0.9	- 4.1	- 4.6			4.8			3.9
	ΔY	- 5.1	3.1	6.6	10.8			4.7			2.7
	ΔZ	- 5.1	- 5.6	- 3.3	- 7.6			5.1			4.3
1030	ΔX	-15.5	- 0.1	- 4.4	- 5.8			8.6			7.9
	ΔY	4.5	- 0.8	11.3	1.2			5.0			4.0
	ΔZ	- 3.6	5.5	$\boxed{-11.9}$	1.1			5.1			4.1
1032	ΔX	- 4.6	-13.3	-72.0	-72.3			69.5			69.0
	ΔY	-91.7	-75.4	-99.5	-94.1			98.5			82.6
	ΔZ	53.7	26.5	14.1	9.4			38.3			23.0
1034	ΔX	- 4.6	2.4	- 8.0	1.4	- 1.7	- 3.9	4.2		7.0	3.7
	ΔY	6.9	- 4.7	0.2	- 6.3	4.7	- 3.7	5.1		7.0	4.4
	ΔZ	3.6	3.2	- 7.6	- 1.1	- 9.7	6.4	4.9		7.0	4.0
1042	ΔX	2.9	0.8	- 8.1	- 6.8	- 9.0	- 6.9	4.8		7.0	4.0
	ΔY	- 1.8	- 1.1	6.9	7.9	2.4	2.0	4.5		7.0	3.0
	ΔZ	- 0.9	- 3.3	0.7	- 1.2	1.6	- 0.6	4.8		7.0	3.9
7036	ΔX	- 6.4	- 1.7	1.3	0.3	6.4	- 1.1	4.9		7.0	4.4
	ΔY	- 0.8	6.0	- 2.4	- 4.2	1.1	1.2	5.1		7.0	3.3
	ΔZ	- 9.2	- 4.9	- 0.1	2.7	2.8	- 1.7	5.7		7.0	3.9

Table 5 continued

		NA 8 - NA 6		NA 8 - GSFC		NA 8 - SAO69					
		Transf. 3 Param.	Transf. 7 Param.	Transf. 3 Param.	Transf. 7 Param.	Transf. 3 Param.	Transf. 7 Param.	σ_{NA6}	σ_{GSFC} (mean)	σ_{SAO}	σ_{NA8}
7037	ΔX	- 3.8	- 0.2	- 2.0	- 0.3	- 1.9	- 3.4	2.9		7.0	2.3
	ΔY	2.2	- 0.8	- 3.6	- 6.6	1.9	- 2.7	2.9		7.0	2.7
	ΔZ	0.8	- 0.0	- 1.4	1.1	- 1.5	3.2	2.9		7.0	2.9
7039	ΔX	16.0	2.8	7.2	6.6	- 6.7	1.8	8.8		10.0	7.3
	ΔY	31.2	33.0	-25.7	-17.6	-21.2	-12.8	6.2		10.0	4.5
	ΔZ	21.7	17.6	- 1.0	- 9.3	- 0.1	- 8.0	6.2		10.0	5.1
7040	ΔX	17.5	- 0.2	14.0	9.7	5.1	10.5	9.0		10.0	7.3
	ΔY	-35.6	-22.1	-27.3	-14.2	-27.8	-10.0	5.9		10.0	3.6
	ΔZ	11.3	11.7	15.9	4.4	25.8	4.4	7.6		10.0	6.9
7045	ΔX	- 7.9	2.1	8.2	9.2	13.3	6.2	6.0		9.0	5.4
	ΔY	7.0	1.4	- 0.8	- 7.9	4.7	- 2.5	5.2		9.0	3.9
	ΔZ	- 5.2	- 2.3	- 6.3	1.5	-13.4	- 2.2	5.2		9.0	4.1
7072	ΔX	5.3	- 0.7	- 0.7	- 1.2			5.4			4.4
	ΔY	- 4.3	3.4	11.0	15.7			5.1			3.1
	ΔZ	- 4.9	- 5.9	0.2	- 4.7			5.4			4.5
7076	ΔX	8.3	- 2.6	5.9	2.6	- 3.0	- 2.0	7.0		10.0	5.9
	ΔY	-27.4	-12.6	-20.3	-11.3	- 1.8	11.1	7.5		10.0	4.3
	ΔZ	7.8	9.5	24.1	16.2	34.0	13.8	8.4		10.0	7.7

3. PERSONNEL

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Evelyn E. Rist, Technical Assistant, full time

4. TRAVEL

Ivan I. Mueller
Key Biscayne Florida, October 5-8, 1971
To attend NOAA/NASA/NAVY Conference.

Ivan I. Mueller
Greenbelt, Maryland, November 17-19, 1971
Discussions with personnel at NASA, Goddard Space Flight Center regarding project reporting.

Ivan I. Mueller
San Francisco, California, December 5-7, 1971
Present two papers at the Annual Fall Meeting of the American Geophysical Union.

5. REPORTS PUBLISHED TO DATE

OSU Department of Geodetic Science Reports published under Grant No. NSR 36-008-003:

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by Hans D. Preuss
April, 1966

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by Ivan I. Mueller
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100 Preprocessing Electronic Satellite Observations
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110 Investigations into the Utilization of Passive Satellite Observational Data
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125 The North American Datum in View of GEOS I Observations
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140 SECOR Observations in the Pacific
by Ivan I. Mueller, James P. Reilly, Charles R. Schwarz, Georges Blaha
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147 Gravity Field Refinement by Satellite to Satellite Doppler Tracking
by Charles R. Schwarz
December, 1970

148 Inner Adjustment Constraints with Emphasis on Range Observations
by Georges Blaha
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150 Investigations of Critical Configurations for Fundamental Range Networks
by Georges Blaha
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The following papers were presented at various professional meetings:

"Report on OSU participation in the NGSP"
47th Annual meeting of the AGU, Washington, D. C., April 1966

"Preprocessing Optical Satellite Observational Data"
3rd Meeting of the Western European Satellite Subcommission, IAG, Venice, Italy, May 1967.

"Global Satellite Triangulation and Trilateration"
XIVth General Assembly of the IUGG, Lucerne, Switzerland, September 1967, (Bulletin Geodesique, March 1968).

"Investigations in Connection with the Geometric Analysis of Geodetic Satellite Data"
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"Comparison of Photogrammetric and Astrometric Data Reduction Results for the Wild BC-4 Camera"
Conference on Photographic Astrometric Technique, Tampa, Fla., March 1968.

"Geodetic Utilization of Satellite Photography"
7th National Fall Meeting, AGU, San Francisco, Cal., Dec. 1968.

"Analyzing Passive-Satellite Photography for Geodetic Applications"
4th Meeting of the Western European Satellite Subcommission, IAG, Paris, Feb. 1969.

"Sequential Least Squares Adjustment of Satellite Trilateration"
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8th National Fall Meeting of the AGU, San Francisco, Cal., Dec. 1969 and GEOS-2 Review Meeting, Greenbelt, Md., June 1970 (Bulletin Geodesique, June 1970).

"Experiments with SECOR Observations on GEOS-I"
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"Experiments with Wild BC-4 Photographic Plates"
GEOS-2 Review Meeting, Greenbelt, Md., June 1970.

"Experiments with the Use of Orbital Constraints in the Case of Satellite Trails on Wild BC-4 Photographic Plates"
GEOS-2 Review Meeting, Greenbelt, Md., June 1970.

"GEOS-I SECOR Observations in the Pacific (Solution SP-7)"
National Fall Meeting of the American Geophysical Union, San Francisco, California, December 7-10, 1970.

"Investigations of Critical Configurations for Fundamental Range Networks"
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"Gravity Field Refinement by Satellite to Satellite Doppler Tracking"
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"Separating the Secular Motion of the Pole from Continental Drift - Where and What to Observe?"
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"Scaling the SAO-69 Geometric Solution with C-Band Radar Data (Solution SC 11)"
Annual Fall Meeting of the American Geophysical Union, San Francisco, California, December 6-9, 1971